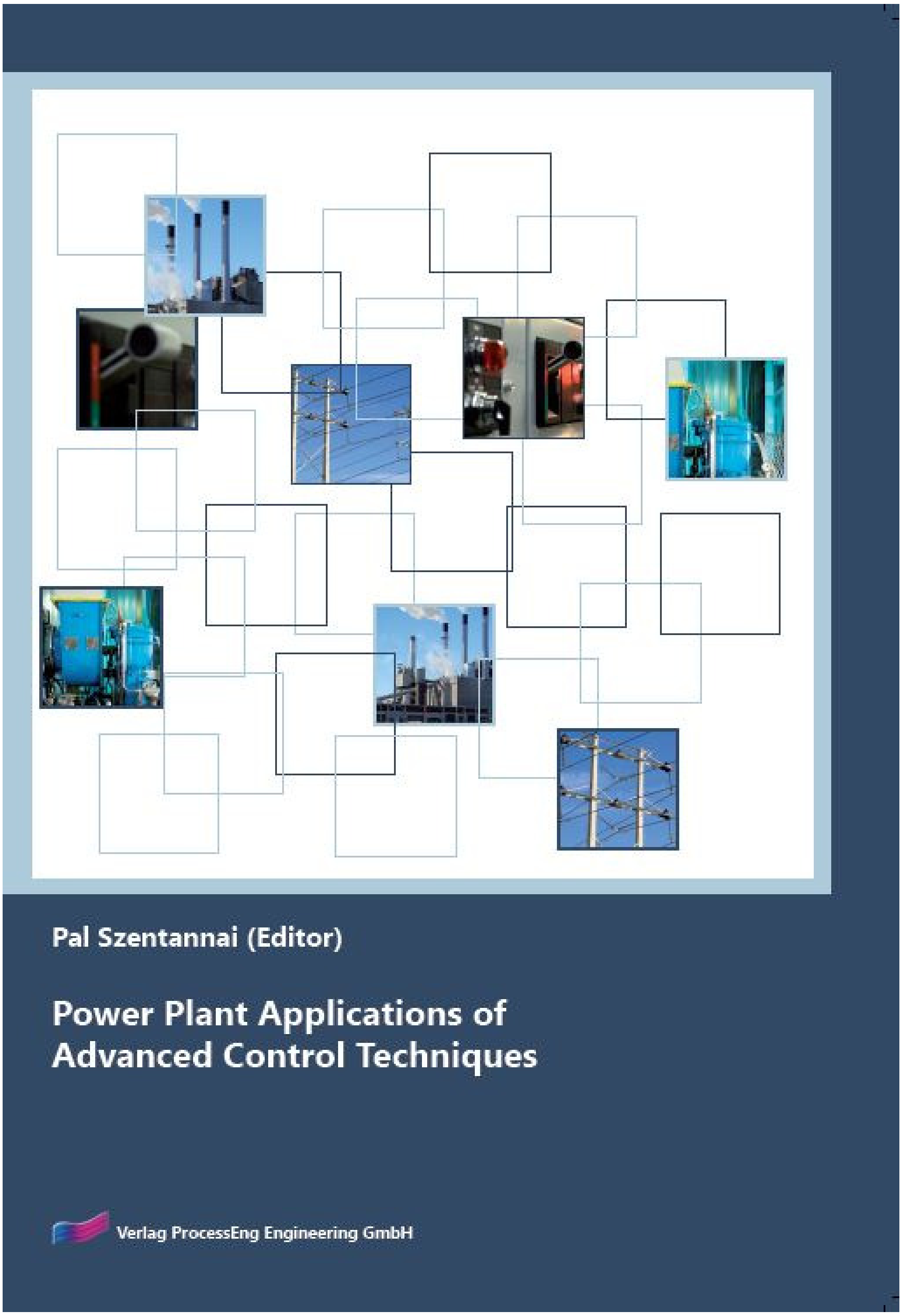

Power Plant Applications of Advanced Control Techniques

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Nonlinear Control Based on Affine Neural Networks: Application to a Solar Power Plant

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1. Introduction

One of the main features of a solar power plant is that its primary energy source, the solar radiation, cannot be manipulated by the control system. Moreover, since the solar radiation changes substantially due to the daily solar cycle and to the atmospheric conditions, significant variations in the dynamics of the plant are observed. Therefore, it is difficult to achieve a satisfactory performance over the whole operating range by using conventional non-adaptive linear control strategies. In fact, using a conventional approach the design of a controller usually demands an accurate analytical model of the process under consideration. However, in real world applications, mainly due to the processes complexity, hard nonlinearities or unmodelled dynamics, the obtained analytical models do not reflect the true physical properties of the process. In such cases, as a result of model/plant mismatch, poor performance of the control system can occur. An alternative is to develop experimental models based on observed input and output data.

The ability of neural networks to learn and generalize based on the input-output behavior of a given process has a great impact. Their universal approximation and generalization properties, and the ability to adjust online their parameters, allow neural networks to answer two of the main challenges for which conventional techniques present serious limitations [1] [2]: *i*) generality and precision in modeling problems and *ii*) adaptation capabilities to time-varying dynamics.
Basically, neural networks can be classified as static (feedforward) and dynamic (recurrent). Dynamic or recurrent neural networks (RNN) were first introduced in [3] and have been developed in other works [4], [5], [6], [7]. Due to their intrinsic ability to incorporate time (involving dynamic elements and internal feedback), RNN structures have advantages with respect to static neural networks for modeling dynamic processes. Moreover, RNN are in a standard form and present a lower order compact structure, making them ideal candidates to be incorporated within model based control schemes [8]. These reasons justify their use in this work.

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An RNN affine structure is proposed here, as a compromise between two main goals: *i*) the development of generic structures, appropriate to modeling nonlinear processes and *ii*) effectiveness for both parameter estimation and control design procedures.
Concerning the neural network training, and taking into account the available data, a previous parameterization for the network is obtained. Subsequently, an online learning is further considered to mitigate the discrepancies between the output of the neural model and current output of the system during operation. This task is carried out by considering the specific state space form of the recurrent neural network, thus enabling a joint estimation of parameters and states. This is carried out based on a dual Kalman filter approach, along with Lyapunov stability analysis for the weights updating.

Moreover, the problem of order estimation for state–space neural networks is addressed as well. The proposed approach is based on subspace projections and on the formal specificity of the nonlinear model structure. The involved matrices are generated using input/output data collected from the system and the order extracted by means of a singular value decomposition (SVD) applied to a non–orthogonal space projection.
Concerning control strategies, there are two main approaches in which neural models can be incorporated [9]: *i*) the neural network is itself a controller (in this case the most common strategy consists in learning the plant's inverse) or *ii*) the network is used as model for the system, being the controller designed through a model based control approach. Although the use of a neural network to learn the plant's inverse is theoretically a viable option, in practice the estimated inverse neural model may not be well defined or stable [10]. Mainly for this reason, the second mentioned control strategy, analogous to a conventional indirect strategy, was the approach followed in this work. Thus, assuming the approximation properties, the recurrent neural model replaces the plant, resulting in a nonlinear control problem suitable to be solved by a nonlinear control technique. Among nonlinear control techniques, the output regulation (OR), which is proposed in this work, enables to derive a control law such that the closed loop system is stable and the tracking error converges to zero.

This methodology was applied to the temperature control of a distributed collector field of a solar power at Plataforma Solar de Almeria, in the south of Spain. This book chapter is organized as follows: in section 2 the solar power plant is presented and some applied control strategies are reviewed. Section 3 introduces the proposed control scheme. In particular the affine recurrent neural network architecture and the respective offline and online learning laws are described. Additionally, the subspace projection approach regarding the complexity management is presented, and the output regulation control strategy described. Section 4 provides some insights on how the proposed control scheme can be applied to the solar power plant. Section 5 presents the experimental results and in section 6 some conclusions are drawn.

2. The Solar Power Plant

2.1 Distributed Solar Collector Field

The Acurex distributed solar collector field at Plataforma Solar de Almeria (PSA) is located at the desert of Tabernas, in south of Spain. For an extensive description the reader is referred to López, [11]. The field consists of 480 distributed solar collectors arranged in 20 rows, which form 10 parallel loops. Each loop is 172 m long and the total aperture surface is $2672\ m^2$, enabling to provide 1.2 MW peak of thermal power. A schematic diagram and a partial picture of the plant are shown, respectively in Figure 1 and Figure 2.

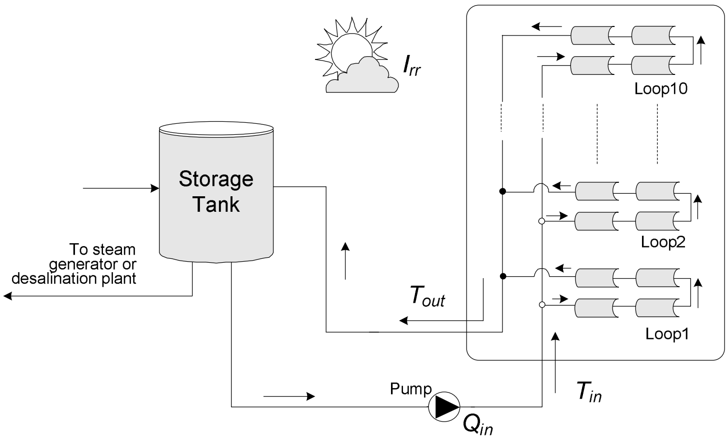


Figure 1: *Schematic diagram of the Acurex field.*

Figure 2: *Partial picture of the Acurex field.*

The cold inlet synthetic oil is collected from the bottom of the storage tank and is passed through the field by means of a pump at the field inlet. After having picked up the heat transferred from the tube walls the heated fluid is fed to the storage tank to be used for electrical energy generation or feeding a heat exchanger of the desalination plant. The fluid used for heat transmission is the Santotherm 55, which is a synthetic hydrocarbon with a maximum film temperature of 318 ºC and an auto ignition temperature of 357 ºC. Therefore 300 ºC is set as the maximum temperature allowed. If the system reaches this temperature in any loop the collectors are sent, for safety reasons, into desteer. Each collector uses parabolic mirrors to concentrate the radiation in a receiver tube being the field also provided with a sun tracking system that causes the mirrors to revolve around an axis parallel to that of the pipe. The manipulated variable in the plant is the oil flow rate, Q_{in}, being the main goal to regulate the outlet field oil temperature, T_{out}, at a desired value, T_{ref} (Figure 2). The main disturbances are the solar radiation, I_{rr}, and the inlet oil temperature, T_{in}. The pump has a maximum flow rate of 12 l/s and the lowest flow rate permitted is 1.6 l/s. However, for safety reasons the flow rate is set between 2 l/s and 9 l/s. In case of pump stop the field goes automatically into desteer to avoid overheating of the oil in the parabolic trough loops.

2.2 Control Strategies Applied to the Solar Power Plant

As stated, one of the main features of this plant is that its primary energy source, the solar radiation, cannot be manipulated by the control system. Moreover, since the solar radiation changes substantially throughout the plant operation, due to the daily solar cycle, atmospheric conditions such as the cloud cover, humidity and air transparency, significant variations in the dynamics of the field (e.g. the response rate and time delay) are observed, corresponding to different operating conditions. Therefore, it is difficult to obtain a

satisfactory performance over the whole operating range with a sole linear fixed controller. One possibility to overcome these issues is to use adaptive control schemes on the basis of local linear models, which can capture dynamics variations during the operation. In this context, Camacho et al [12] and Rubio et al [13] proposed a self tuning scheme involving a PI (Proportional-Integral) controller. Pickhardt and Silva [14] proposed an adaptive predictive control scheme, using an input output linear model of the plant. A feature of the distributed solar collector field is that the main disturbances, the solar radiation and the inlet oil temperature, are measurable. Following this idea Coito et al [15] presented simulation and experimental results concerning the design of a predictive controller (MUSMAR), and Cardoso et al [16] presented a fuzzy supervisory strategy that takes into account these disturbances.
Another possible alternative could be the commissioning of a switching controller using different models of the plant for different operating points. Henriques et al [17] proposed a control strategy based on a PID control design with fuzzy switching supervisory. Combining the potentialities of neural networks for approximation purposes with the well known theory and widespread application of PID techniques to this type of processes, Henriques et al [18] developed and applied successfully a simple but effective structure that uses a feedforward neural network for scheduling a PID controller, from a bank of PID controllers, previously tuned for the most relevant operating point.
Other authors suggested intelligent control techniques (Rubio et al, [19]), such as fuzzy systems (Flores et al, [20]), neuro-fuzzy and evolutionary programming (Kharaajoo, [21]) or neural networks (Lalot, [22]). Using neural network methodologies Gil et al [23] proposed a feedback linearization control scheme where a recurrent neural network is employed as a predictive model; furthermore, given the neural model mismatches, the control system is also provided with a steady offset compensation, being an internal model control strategy considered in their methodology.
Other authors have followed a modeling approach, in order to derive predictors, to be used in nonlinear predictive control schemes. In this context Gil et al [24], have proposed the application of a nonlinear adaptive constrained model based predictive control scheme to the distributed collector field. Their methodology exploits the intrinsic nonlinear modeling capabilities of nonlinear state space neural networks and their online training by means of an unscented Kalman filter. Carrillo et al, [25], [26] proposed the application of a nonlinear control strategy, designated as nonlinear extended prediction self adaptive control, where a Smith Predictor is used together with a model of the plant. Arahal et al [27] proposed the use of neural networks to modeling the plant and their incorporation in a nonlinear generalized predictive control scheme. Kharaajoo [21], proposed the application of an intelligent predictive control strategy. A neuro-fuzzy scheme is used to characterize the future behavior of the plant over a certain prediction horizon, while an evolutionary programming algorithm computes the control action, by means of an optimization procedure.
Camacho et al [28], [29] presented an extensive survey on automatic control approaches that have been applied to the distributed collector field of the PSA during the last years. A classification of the modeling and control approaches is proposed in order to expose the main features of each strategy.

3. Nonlinear Control Approach

As mentioned above, an important characteristic of a solar power plant is that the primary energy source, the solar radiation, cannot be manipulated. It varies throughout the day, causing changes in plant dynamics conducting to distinct main several operating points. Thus, the control design of such processes is commonly addressed by considering a specific controller for each local conditions. In this context, traditional PID controllers have some well known advantages, such as the performance provided for some particular nominal operating points and their industrial widespread. Moreover, they are simple to implement and they can successfully control many industrial processes with distinct specifications.

Although it is possible to design an acceptable PID controller for each operating condition of the solar power plant, due to the underlying complex behavior, poor transient performance is achieved in other operating conditions. To deal with these constraints, an indirect adaptive control methodology is proposed. This methodology has the ability to recursively adjust the model parameters, at each sampling time, based on specific plant conditions. Then, based on the adaptive model, an appropriate controller can be designed. In particular, due to the non-linear characteristics of the plant, non-linear control strategies are expected to outperform linear approaches.

3.1 Affine Recurrent Neural Networks

Given the approximation capabilities of RNN [8] it is assumed that there exists an affine RNN, described by (1) and shown in Figure 3 that is able to describe the plant dynamics.

$$\begin{aligned} x_n(k+1) &= A\ x_n(k) + B\ u(k) + D\ \sigma\left(x_n(k)\right) \\ y_n(k+1) &= C\ x_n(k+1) \end{aligned} \qquad (1)$$

where the vector $x_n(k) \in \Re^n$ is the output of the hidden neural layer, also known as the network hyper-state, $y_n(k) \in \Re^{ny}$ is the network output and $u(k) \in \Re^{nu}$ and $y(k) \in \Re^{ny}$ are, respectively, the input and output vectors, at discrete-time k. $A \in \Re^{n \times n}$, $B \in \Re^{n \times nu}$, $C \in \Re^{ny \times n}$ and $D \in \Re^{n \times n}$ are interconnection matrices. The activation function, $\sigma(\cdot)$, is chosen as the hyperbolic tangent function.

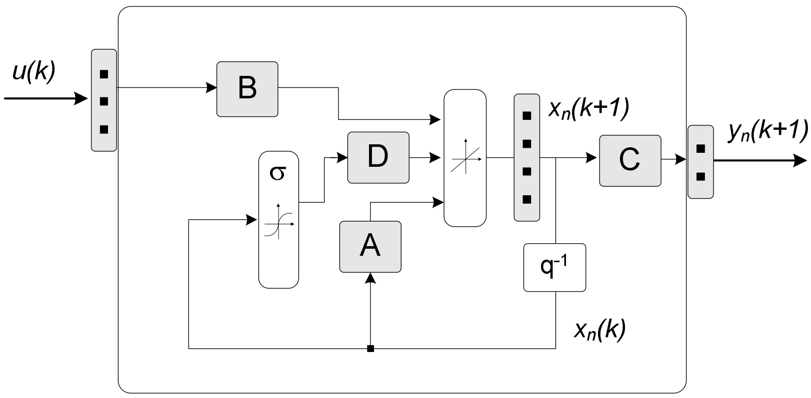


Figure 3: *Affine recurrent neural network.*

It is important to stress that the network does not behave in the strictest sense as an observer. In fact, it is not expected that the true states of the system are correctly determined, but that a state space representation for the system is achieved. The main goal is that the plant identification error, $e_i(k) \in \Re^{ny}$, defined as the difference between the real process output, $y(k)$, and of the neuronal output, $y_n(k)$, converges to zero:

$$\lim_{k \to \infty} e_i(k) = \lim_{k \to \infty} \left(y(k) - y_n(k)\right) = 0 \qquad (2)$$

3.2 Learning Methodology

The learning methodology consists in a prior offline learning and an a *posteriori* online adjusting of the model's parameters.

3.2.1 Offline learning

As pointed out by Suratgar [30] the Levenberg-Marquardt algorithm is more efficient than other techniques when the network contains no more than a few hundred parameters. Due to its effectiveness this algorithm is applied to offline training. This initial training phase provides the interconnection matrices A, B, C and D.

3.2.2 Online learning

The online learning law for estimating the RNN parameters is based on a dual Kalman filter, where both the estimated hyper-state $x_s(k)$ and the network parameters $W(k)$ are updated. The schematics showing the simultaneous estimation of the state vector and parameters is depicted in Figure 4.

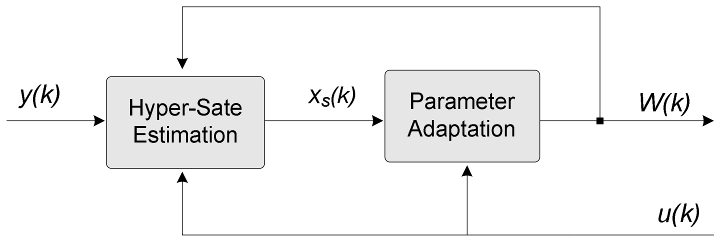


***Figure 4**: Joint estimation of parameters and states.*

Let assumes that A and C do not change (offline evaluated) and only matrices B and D are updated online, at each time step k. The online learning problem aims at evaluating an estimation $x_s(k) \in \Re^n$ for the hyper-states $x_n(k)$ and to evaluate the parameter vector $W(k) \in \Re^{nw}$, defined at each instant by (3), where n_w is the number of parameters to be adapted (matrices B and D), $n_w = n \times n_u + n \times n$.

$$W(k) = \left[\vec{B}(k)\ \vec{D}(k) \right] \tag{3}$$

Vectors $\vec{B}(k) \in \Re^{nb}$ and $\vec{D}(k) \in \Re^{nd}$ are composed of the elements of matrices B and D ($n_b = n \times n_u$ and $n_d = n \times n$).

For the hyper-state estimation, a nonlinear observation methodology was developed and implemented. Namely, the observation results proposed by Thau [31] for continuous nonlinear systems are extended to the proposed recurrent nonlinear model, assuming the pair (A, C) observable.

Concerning the second problem, several training algorithms have been proposed to recursively adjusts the neural network's parameters. Basic examples are the real time recurrent algorithm [32], the dynamic backpropagation [33], and the backpropagation trough time [34]. Recently, other advanced methods have been proposed, [35], [36]. Basically, the training algorithm iteratively update the weights W according to (4).

$$W(k+1) = W(k) + \Delta W(k) \tag{4}$$

However, with respect to the updating law, few stability results have been presented. In the following, a stable online updating law based on the *Lyapunov* stability theory is proposed, analogous to a result established in the context of control [37]. Additionally, it is assumed that A is a *Hurwitz* matrix, that is, its eigenvalues lie in the unitary circle.

Hyper-state estimation

The recurrent network consists of a hybrid model where the nonlinear part (1) is due to the hyperbolic tangent, which is a *Lipschitz* function (nonlinearity with constant ς).

$$D\ \sigma\left(x_n(k)\right)=\varsigma\left(x_n(k)\right) \tag{5}$$

Taking into account this nonlinearity, the state observation procedure exploites the extension of the continuous-time results proposed by Thau, [31]. The observer is defined as follows [18], similar to an Luenberger observer [38].

$$\begin{aligned} x_s(k+1) &= Ax_s(k)+Bu(k)+D\sigma\left(x_s(k)\right)+L\left(y_n(k)-Cx_s(k)\right) \\ y_s(k) &= Cx_s(k) \end{aligned} \tag{6}$$

Assuming that the pair (A,C) is observable, there exists a matrix $L\in\Re^{n\times ny}$ (observer gain) that places the eigenvalues of matrix $A_o\in\Re^{n\times n}$, defined as *(7)*, at desired locations.

$$A_o = A-LC \tag{7}$$

In these conditions the observation error converges to zero if the following relation holds [18].

$$\left|D\tau\right| < -\left|A_o\right| + \sqrt{\left|A_o\right|^2+\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}} \tag{8}$$

$\lambda_{\min}(Q)$ and $\lambda_{\max}(P)$ are, respectively, the minimum and maximum eigenvalues of matrices Q and P and $\left|M\right|$ is the quadratic norm of matrix M. $P\in\Re^{n\times n}$ and $Q\in\Re^{n\times n}$ are positive definite symmetric matrices for which the discrete-time Lyapunov equation (9) holds.

$$A_o{}^T PA_o - P = -Q \tag{9}$$

Parameters Adaptation

Following the estimation of the states, the next step involves the adaptation of the network's parameters. Analogous to a dual Kalman filter, the states of the neural model $x_n(k)$ are assumed to be known and thus the problem consists in updating the network matrices $B(k)$ and $D(k)$ such that the observation error converges to zero. Assuming the observer described by:

$$\begin{aligned} x_s(k+1) &= Ax_s(k)+B(k)u(k)+D(k)\sigma\left(x_s(k)\right) \\ y_s(k) &= Cx_s(k) \end{aligned} \tag{10}$$

and the affine neural network (1) it is possible to write (11).

$$\varepsilon(k+1)=Ax_n(k)-Ax_s(k)+\left(B-B(k)\right)u(k)+D\sigma\left(x_n(k)\right)-D(k)\sigma\left(x_s(k)\right) \tag{11}$$

Assuming the estimation of the states, thus $\sigma\left(x_n(k)\right)\approx\sigma\left(x_s(k)\right)$

$$\varepsilon(k+1)=Ax_n(k)-Ax_s(k)+\left(B-B(k)\right)u(k)+\left(D-D(k)\right)\sigma\left(x_n(k)\right) \tag{12}$$

Let the matrix errors $\tilde{B} \in \Re^{n \times nu}$ and $\tilde{D} \in \Re^{n \times n}$ be respectively given by (13).

$$\begin{aligned} \tilde{B}(k) &= B - B(k) \\ \tilde{D}(k) &= D - D(k) \end{aligned} \tag{13}$$

Then, from (12), (13) and follows (14),

$$\varepsilon(k+1) = A\ \varepsilon(k) + \tilde{B}(k)\ u(k) + \tilde{D}(k)\ \sigma\left(x_n(k)\right) \tag{14}$$

or,

$$\varepsilon(k+1) = A\ \varepsilon(k) + \varphi(k)\ \tilde{W}(k) \tag{15}$$

where $\varphi(k) \in \Re^{n \times nw}$ is an information matrix depending on the state $\sigma\left(x_n(k)\right)$ and on the input $u(k)$, given by,

$$\varphi(k) = \begin{bmatrix} u(k)^T & \varnothing_u & \dots & \varnothing_u & \vdots & \sigma\left(x_n(k)\right)^T & \varnothing_x & \dots & \varnothing_x \\ \dots & \dots & \dots & \dots & \vdots & \dots & \dots & \dots & \dots \\ \varnothing_u & \dots & \varnothing_u & u(k)^T & \vdots & \varnothing_x & \dots & \varnothing_x & \sigma\left(x_n(k)\right)^T \end{bmatrix} \tag{16}$$

where the vectors $\varnothing_u(k) \in \Re^{nu}$ and $\varnothing_x(k) \in \Re^{n}$ are composed of zeros. The vector $\tilde{W}(k) \in \Re^{nw}$ is composed of the parameters errors to be estimated, defined by (17).

$$\tilde{W}(k) = \begin{bmatrix} \vec{\tilde{B}}(k) & \vec{\tilde{D}}(k) \end{bmatrix} \tag{17}$$

Since the difference between two consecutive instants can be written as

$$\Delta\tilde{W}(k) = \tilde{W}(k) - \tilde{W}(k-1) \tag{18}$$

it is possible to relate $\Delta\tilde{W}(k)$ with $\Delta W(k)$ by (19).

$$\Delta\tilde{W}(k) = \left(W^* - W(k)\right) - \left(W^* - W(k-1)\right) = -\Delta W(k) \tag{19}$$

Where W* is a vector composed of the parameters of matrices B and D. Thus, once the parameter vector error $\tilde{W}(k)$ obtained at instant k and since $\tilde{W}(k-1)$ is known, it is possible to evaluate $\Delta\tilde{W}(k)$ and so updating $\Delta W(k)$.

With these definitions in mind, the updating law of weights variations given by

$$\tilde{W}(k) = -M(k)^{-1} \varphi(k)^T P\ A\ C\ e_y(k) \tag{20}$$

ensures the convergence of the observation error $\varepsilon(k)$. The matrix $M(k) \in \Re^{nu \times nw}$ is defined by (21) and $P \in \Re^{n \times n}$ is a positive definite matrix obtained from the discrete-time Lyapunov equation (22).

$$M(k) = \left[I_{nw} + \frac{1}{2} \varphi(k)^T P\ \varphi(k) \right] \tag{21}$$

$$A^T P A - P = -Q \tag{22}$$

$Q \in \Re^{n \times n}$ is a positive definite matrix to be specified by the designer and $I_{nw} \in \Re^{nw \times nw}$ is the identity matrix.

Since the eigenvalues of matrix A are in the closed unitary circle the solution P to equation (22) exists and is unique. P and M are symmetric matrices, thus $P = P^T$ and $M = M^T$. Given that M is a square matrix, with dimension (n_w, n_w), its inversion, to be performed at each discrete-time, should be regarded as a drawback of the algorithm.

Discussion

As mentioned, it is not intended in any case to determine precisely the true states of the plant and, in this sense, the proposed methodology should not be viewed as a true observer. To be precise, only one possible state space description of the plant is obtained. In fact, the strategy followed here provides a procedure that ensures the stability and convergence to zero of the observation error $\varepsilon(k)$. However, since output error is given by $C\varepsilon(k)$, output convergence error is straightforward. The proposed methodology can only be applied if the pair (A, C) is observable and the eigenvalues of matrix A lie within the unitary circle. Therefore, if one considers the neural model as a hybrid structure, composed of a linear and a nonlinear part, then this methodology will only be feasible if the linear part is observable and stable. Considering practical applications, this constraint has to be considered as a limitation of the proposed methodology. In practice, such imposition restricts the initial determination of the network parameters and the class of systems to which this method can be applied.

3.3 Order Estimation

An important issue concerning the generalization capabilities of state–space neural networks is the complexity of the underlying topology. As pointed out elsewhere (see e.g. [39]), neural networks are intrinsically prone to overfitting as a converging result of an excessive number of parameters and the unconstrained minimisation of the empirical loss function.
One method known to control the smoothness of the fit is to add a regularization term to the loss function (see e.g. [40]). Other techniques, which are devoted to selecting the number of hidden-layer neurons, include network pruning and constructive methods (see e.g. [41]), statistical approaches, such as the Network Information Criteria (NIC) [42] and methods based on the application of Vapnik–Chevornenkis dimension [43]. Using a global optimization approach based on the advantages of simulated annealing and tabu search T. Ludermir et al [44] have proposed a methodology for simultaneous optimization of weights and topologies with few connections and high generalization capabilities. A different approach was followed in [45] where the number of hidden layer neurons is selected so as to guarantee a given approximation order, depending on the number of inputs provided to the neural network and the desired approximation order. Based on the singular value decomposition (SVD) of the output activation matrix along with constructive heuristics Teoh et al [46] proposed a methodology to estimate the appropriate number of hidden layer neurons.
In this book chapter it is proposed a conceptually different approach to deal with the problem of selecting the appropriate number of neurons to be inserted in the hidden layer of the affine state–space neural network structure. The underlying idea relies, essentially, on the specificity of the model structure and is based on the application of the SVD to a given subspace projection.
Let consider the affine state-space model given by (1) and assume that the nonlinear term associated with the sigmoidal activation function performs a spatio–temporal compensation to the linear part contribution provided by a linear model. Thus, by removing the nonlinear term it is possible by using the SVD applied to a given subspace projection (see e.g. [47]) to come up with a vector basis for a linear state–space realization. The dimension of this vector basis can be regarded as a bound to the number of hidden layer neurons. This stems from

the fact that *i*) the order of each affine state–space realization is inexorably related to the number of hidden layer neurons and *ii*) the linear model complexity, measured by the model's order, is likely to be higher than the affine state–space topology in order to capture the relevant system's dynamics embedded in the data set.

A finite dimensional discrete–time invariant linear system can be represented in the state–space form as

$$\begin{aligned} x(k+1) &= A\,x(k) + B\,u(k) + \eta(k) \\ y(k) &= C\,x(k) + \vartheta(k) \end{aligned} \tag{23}$$

where $\eta \in \Re^n$ and $\vartheta \in \Re^n$ are unobserved Gaussian distributed, zero mean, white noise sequences, and assuming the system (23) satisfies the orthogonality property (24) and the estimation data are ergotic.

$$E\left[\begin{pmatrix} x(k) \\ u(k) \end{pmatrix} \begin{pmatrix} \eta^{\mathrm{T}}(k) & \vartheta^{\mathrm{T}}(k) \end{pmatrix}\right] = 0 \tag{24}$$

3.3.1 Subspace Projection

Let the future input block Hankel matrix $U_{\mathrm{f}} = U_{i|2i\text{-}1}$ be defined as

$$U_{\mathrm{f}} \triangleq \begin{bmatrix} u(i) & u(i+1) & \cdots & u(i+j-1) \\ u(i+1) & u(i+2) & \cdots & u(i+j) \\ \vdots & \vdots & \cdots & \vdots \\ u(2i-1) & u(2i) & \cdots & u(2i+j-2) \end{bmatrix} \tag{25}$$

and the future output block Hankel $Y_{\mathrm{f}} = Y_{i|2i\text{-}1}$ given by

$$Y_{\mathrm{f}} \triangleq \begin{bmatrix} y(i) & y(i+1) & \cdots & y(i+j-1) \\ y(i+1) & y(i+2) & \cdots & y(i+j) \\ \vdots & \vdots & \cdots & \vdots \\ y(2i-1) & y(2i) & \cdots & y(2i+j-2) \end{bmatrix} \tag{26}$$

Where the number of block rows *i* should be larger than the "expected" maximum order of the system $(n \ll i)$ and $j \to \infty$.

Definition 1 (Oblique projection): The oblique projection of the row space of $\mathrm{A} \in \mathbb{R}^{p\times j}$ along the row space of $\mathrm{B} \in \mathbb{R}^{q\times j}$ on the row space of $\Gamma \in \mathbb{R}^{q\times j}$ is given by

$$\mathrm{A}/_{\mathrm{B}}\,\Gamma = \mathrm{A}\begin{bmatrix} \Gamma^{\mathrm{T}} & \mathrm{B}^{\mathrm{T}} \end{bmatrix} \left[\begin{pmatrix} \Gamma\Gamma^{\mathrm{T}} & \Gamma\mathrm{B}^{\mathrm{T}} \\ \mathrm{B}\Gamma^{\mathrm{T}} & \mathrm{B}\mathrm{B}^{\mathrm{T}} \end{pmatrix}^{\dagger}\right]_{1:r} \Gamma \tag{27}$$

Definition 2 (Persistency of excitation): A signal $u \in \mathbb{R}^m$ fulfils the condition of persistent excitation of order $2i$ if the input covariance matrix $[U_{0|2i-1} \quad U_{0|2i-1}^{\mathrm{T}}]j^{-1}$ is of full rank.

Theorem 1 (Main subspace identification theorem [48]): Assuming that:

i) The deterministic input is uncorrelated with the process noise and measurement noise;

ii) The process noise and the measurement noise are not identically zero;
iii) The exogenous input is persistently exciting of order $2i$;
iv) The data set is large enough ($j \to \infty$).

Then:

i) The projection Π_i can be defined as the oblique projection of the row space of Y_f on the past input/output row space along the row space of U_f;

$$\Pi_i = Y_\mathrm{f} /_{U_\mathrm{f}} M_\mathrm{p} \tag{28}$$

with $M_\mathrm{p} \triangleq \begin{pmatrix} U_{0|i-1} \\ \hline Y_{0|i-1} \end{pmatrix}$.

ii) The order of the system is given by the number of non-zero singular values of Π_i.

The rank of Π_i, which is given by the cardinality of the non–zero singular values, provides an estimate to the linear state space model order. A straightforward way to retrieve the singular values of Π_i is by using the SVD of the underlying projection, under the strict conditions of theorem 1.

Theorem 2 (The SVD theorem [49]): Let the projection matrix $\Pi \in \mathbb{R}_r^{p\times q}$ $[\mathbb{C}_r^{p\times q}]$. Then there exist orthogonal (unitary) matrices $U \in \mathbb{R}^{p\times p}$ $[\mathbb{C}^{p\times p}]$ and $V \in \mathbb{R}^{q\times q}$ $[\mathbb{C}^{q\times q}]$ such that,

$$\Pi = U\,\Sigma\,V^\mathrm{T} \ \ [\Pi = U\,\Sigma\,V^\mathrm{H}] \tag{29}$$

where $\Sigma = \begin{bmatrix} S & 0 \\ 0 & 0 \end{bmatrix}$ and $S = diag(\sigma_1,\ldots,\sigma_r)$, with $\sigma_1 \ge \sigma_2 \cdots \ge \sigma_r > 0 = \sigma_{r+1} = \cdots = \sigma_q$.

When applying this approach to finite data sets ($j \ll \infty$) and/or to data collected from nonlinear systems, even in the case of infinite data sets, the underlying oblique projection $\tilde{\Pi}_i$ will have, in most cases, full rank ($q = p \cdot i$), regardless the number of row blocks *i*. In these cases it is necessary to carry out an additional complexity reduction by finding the cardinality of dominant singular values, instead of relying on the dimension of nonzero singular values.

3.3.2 Numerical Rank

Assume that the full rank degeneracy is derived from additive contamination of an unknown “ideal” projection Π_i, considering an infinite noise-free data set sampled from a linear system, that is,

$$\tilde{\Pi}_i = \Pi_i + \Theta_i \tag{30}$$

where Θ_i is an unknown perturbation matrix.

The problem of finding the numerical rank of $\tilde{\Pi}_i$ can be tackled by defining a tolerance ε, related to its “dominant” singular values, such that $\tilde{\Pi}_i$ is numerically defective in rank if, to within this tolerance, it is near a defective matrix. Specifically, $\tilde{\Pi}_i$ has $\varepsilon-$rank $r < n$ with respect to the norm $\|\cdot\|$ if

$$r = \inf\left\{\text{rank}\left(\Pi_i\right) : \left\|\tilde{\Pi}_i - \Pi_i\right\| \le \varepsilon\right\} \quad (31)$$

By imposing a bound δ on the threshold such that the numerical rank remains at least equal to r the $\varepsilon-\text{rank}$ formulation can be rewritten as follows,

$$\varepsilon < \delta \le \sup\left\{\eta : \left\|\tilde{\Pi}_i - \Pi_i\right\| \le \eta \Rightarrow \text{rank}\left(\Pi_i\right) \ge r\right\} \quad (32)$$

Definition 3 (Numerical rank [49]): A matrix $\mathcal{M}$ has numerical rank $\left(\delta, \varepsilon, r\right)$ with respect to the norm $\|\cdot\|$ if δ, ε and r satisfy (31) and (32).

When the norm is the Frobenius norm $\|\cdot\|_F^2$ the problem of finding the numerical rank can be solved in terms of singular values, that is,

$$\inf_{\text{rank}(\Pi_i)\le r} \left\|\tilde{\Pi}_i - \Pi_i\right\|_F^2 = \sigma_{r+1}^2 + \sigma_{r+2}^2 + \cdots + \sigma_n^2 \quad (33)$$

Theorem 3 [49]): Let $\tilde{\sigma}_1 > \tilde{\sigma}_2 > \cdots > \tilde{\sigma}_n$ be the singular values of $\tilde{\Pi}_i$. Then the projection matrix $\tilde{\Pi}_i$ has numerical rank $\left(\delta, \varepsilon, r\right)_F$ if and only if,

$$\tilde{\sigma}_r^2 + \tilde{\sigma}_{r+1}^2 + \cdots + \tilde{\sigma}_q^2 \ge \delta^2 > \varepsilon^2 \ge \tilde{\sigma}_{r+1}^2 + \cdots + \tilde{\sigma}_q^2 \quad (34)$$

This theorem states that the presence of small singular values is a sufficient condition for rank deficiency. Since the projection Π_i is assumed as being contaminated with error Θ_i a plausible bound for the tolerance ε^2 can be chosen by means of Schmidt's subspace theorem [50], that is,

$$\tilde{\sigma}_{r+1}^2 + \cdots + \tilde{\sigma}_q^2 \le \left\|\Theta_i\right\|_F^2 \quad (35)$$

Thus the SVD of $\tilde{\Pi}_i$ reveals the numerical rank in the sense that the sum of squares of its $q-r$ smallest singular values is bounded by the Frobenius norm of the contamination matrix Θ_i [51]. Therefore, the spaces spanned by $\tilde{U}_1$, $\tilde{U}_2$, $\tilde{V}_1$ and $\tilde{V}_2$ are approximations to the uncontaminated subspaces that are accurate to about $\tilde{\sigma}_r^{-1}\left\|\Theta_i\right\|$. If $\tilde{\sigma}_r$ is large enough, compared to the contamination matrix Θ_i, the problem has a favourable signal to noise ratio and consequently the underlying SVD provides good estimations to the uncontaminated subspaces. Moreover, $\tilde{\sigma}_r$ can be interpreted as a dependable measure of how far, in the $\|\cdot\|$ sense, a matrix is from matrices of lesser rank.

Despite its formal feasibility, the current approach has the implicit drawback of requiring some *a priori* knowledge on the error matrix Θ_i, either under the form of an estimate of a norm of Θ_i or the size of a "typical" element of Θ_i, or even a statistical distribution of the entries of Θ_i.

Another way of defining a threshold for rank deficiency is by restating theorem 3 in terms of spectral norm.

Theorem 4 ([49]): Let $\tilde{\sigma}_1 > \tilde{\sigma}_2 > \cdots > \tilde{\sigma}_n$ be the singular values of $\tilde{\Pi}_i$. Then the projection matrix $\tilde{\Pi}_i$ has numerical rank $\left(\delta, \varepsilon, r\right)_2$ if and only if,

$$\tilde{\sigma}_r \ge \delta > \varepsilon \ge \tilde{\sigma}_{r+1} \quad (36)$$

This theorem states that if there exists a given threshold δ such that $\tilde{\sigma}_r \ge \delta$ and a "gap" $\mu = \delta - \varepsilon$ between consecutive singular values, $\tilde{\sigma}_r - \tilde{\sigma}_{r+1} \ge \mu$, then $\varepsilon-\text{rank}\left(\Pi_i + \Theta_i\right) = r$.

Thus, the numerical rank of $\tilde{\Pi}_i$ can be characterized in terms of sensitivity of the underlying singular values, that is,

$$r = \min_{\|\Theta_i\|_2 \le \varepsilon} \left\{ \text{rank}\left(\Pi_i + \Theta_i\right) \right\} \tag{37}$$

3.3.3 Information Criteria

An alternative approach for managing the model's complexity based on the information embedded in singular values of $\tilde{\Pi}_i$ is by comparing the significance of including additional coordinates in the underlying realization to a given penalty term. This term is chosen such that the parameterization estimates present a number of required properties, in particular, consistency.
Let the general singular value criteria (SVC) be described by the following function [47]:

$$\text{SVC}\left(r, C_\text{T}\right) = \Phi\left(\hat{R}_r\right) + \frac{d\left(r\right)C_\text{T}}{T} \tag{38}$$

where $\hat{R}_r$ is the approximation error due to removed singular values $\tilde{\sigma}_{r+1},\ldots,\tilde{\sigma}_n$, $d\left(r\right)$ denotes the number of parameters associated to a r^{th} order state–space realization, $C_\text{T} > 0$, $C_\text{T}\, T^{-1} \to 0$ is a penalty term punishing high complexity structures, where T refers to the data set length and $\Phi\left(\cdot\right)$ a real valued function.
The estimated order is then obtained as the minimizing argument of (38):

$$\hat{r} = \arg \min_{0<r<n} \text{SVC}\left(r, C_\text{T}\right) \tag{39}$$

Concerning the function $\Phi\left(\hat{R}_r\right)$ Bauer [47] has proposed the squared of Euclidean norm of the contamination matrix

$$\Phi\left(\hat{R}_r\right) = \tilde{\sigma}_{r+1}^2 \tag{40}$$

Concerning C_T several choices have been suggested (see e.g. [52]), although it is recognised that further work is still required to improve the overall contribution of the penalty term. Specifically, $C_\text{T} = \log T$ tends to overestimate the system's order for large data sets, while $C_\text{T} = \hat{p}_{\text{AIC}}^2 \log T$, with $\hat{p}_{\text{AIC}}$ the auto regressive model order given by the Akaike information criterion (AIC) shows high rate of misspecification for small data sets.

3.4 Nonlinear Output Regulation Theory

The output regulation problem for linear systems was first solved by Francis and Wonham, [53]. For nonlinear systems, the fundamental results were introduced by Isidori and Byrnes [37]. They have extended the results of Francis to the general case of time-varying exosystems, and have shown that the linear matrix equations of Francis are particular cases of certain nonlinear partial differential equations, known as *Francis-Isidori-Byrnes* equations.
Basically, the idea of Isidori and Byrnes consists in finding a feedback control law such that it stabilizes the plant around the equilibrium point at the origin with a zero exogenous signal (reference). Consequently, by applying the Center Manifold theory [54], they have presented conditions for closed loop asymptotic stability in case of a small exogenous signal.

3.4.1 Problem Formulation

Given a general multivariable state space discrete-time system

$$\begin{aligned} x(k+1) &= f\left(x(k), u(k), \rho(k)\right) \\ y(k) &= g\left(x(k)\right) \end{aligned} \tag{41}$$

with an additional external variable $\rho(k)$, defined as (42).

$$\rho(k+1) = s\left(\rho(k)\right) \tag{42}$$

This vector $\rho(k) \in \Re^q$ defines the disturbances and/or the reference signal, generated by a so-called exosystem, being $s : \Re^q \rightarrow \Re^q$ is a nonlinear function. The output tracking error, $e_c(k) \in \Re^{ny}$ is defined by $e_c(k) = g(x(k)) - y_d(k)$, considering the desired output $y_d(k) \in \Re^{ny}$ (reference) generated by (43), where $r : \Re^q \rightarrow \Re^{ny}$ is a nonlinear mapping.

$$y_d(k) = r\left(\rho(k)\right) \tag{43}$$

It is assumed that the mappings $f\left(x(k), u(k), \rho(k)\right)$ and $s\left(\rho(k)\right)$ are smooth functions satisfying $f\left(0,0,0\right) = 0$ and $s(0) = 0$. Given this extended system, the problem of asymptotically tracking a reference trajectory is to find a state feedback control law such that the error $e_c(k)$ goes to zero and the whole system is asymptotically stable. Specifically, it is desired to find a controller in the form,

$$u(k) = \gamma\left(x(k), \rho(k)\right) \tag{44}$$

where $\gamma : \Re^{n,q} \rightarrow \Re^{nu}$ is a smooth mapping, satisfying requirements R1 and R2.

R1: The equilibrium point $x(k) = 0$ of dynamics

$$x(k+1) = f\left(x(k), \gamma(x(k)), 0\right) \tag{45}$$

is locally exponentially stable.

R2: There exists a neighborhood of the origin (0,0) such that, for each initial state $\left(x(0), \rho(0)\right)$ the solution of the closed loop system given as,

$$\begin{aligned} x(k+1) &= f\left(x(k), \gamma(x(k), \rho(k)), \rho(k)\right) \\ \rho(k+1) &= s\left(\rho(k)\right) \end{aligned} \tag{46}$$

satisfies the control error condition as follows.

$$\lim_{k \to \infty} e_c(k) = y(k) - y_d(k) = 0 \tag{47}$$

3.4.2 Problem Solution

Solutions to the regulation problem have been presented both in continuous and discrete-time context, Isidori and Byrnes [37] and Castillo-Toledo et al, [60], [61], [62]. However, they have assumed the exact knowledge of the true model and that the systems' states are completely accessible, which is not always viable. One way to overcome these limitations is to exploit the modeling capabilities of the affine state space neural network. In fact, the neural model can be considered as an observer for a nonlinear process along with joint estimation of parameters and states. On the other hand, due to nonlinearities, the derivation of an analytical solution to the regulation problem is, in most cases, impossible. One way to

come up with a solution to the underlying problem is by means of an iterative algorithm, based on the RNN structure. The approach enables the OR problem to be handled as an eigenvalue assignment design, for which is possible to achieve a convergent solution.
In particular, Castillo-Toledo et al [61] have shown that the state feedback discrete-time regulator problem is locally solvable if there exist two mappings $x(k)=\pi\left(\rho(k)\right)$ and $u(k)=\alpha\left(\rho(k)\right)$, with $\pi\left(\rho(0)\right)=0$ and $\alpha\left(\rho(0)\right)=0$, such that the following set of nonlinear difference equations, known as regulation equations, is held.

$$\begin{aligned}\pi\left(s\left(\rho(k)\right)\right)&=f\left(\pi\left(\rho(k)\right),\alpha\left(\rho(k)\right),\rho(k)\right)\\0&=g\left(\pi\left(\rho(k)\right)\right)-r\left(\rho(k)\right)\end{aligned} \tag{48}$$

Once the mappings $x(k)=\pi\left(\rho(k)\right)$ and $u(k)=\alpha\left(\rho(k)\right)$ are evaluated, it is straightforward to show [37] that a particular control law satisfying S1 and S2 is given by (49) and shown in Figure 5.

$$u(k)=\gamma\left(x(k),\rho(k)\right)=\alpha\left(\rho(k)\right)+K\left(x(k)-\pi\left(\rho(k)\right)\right) \tag{49}$$

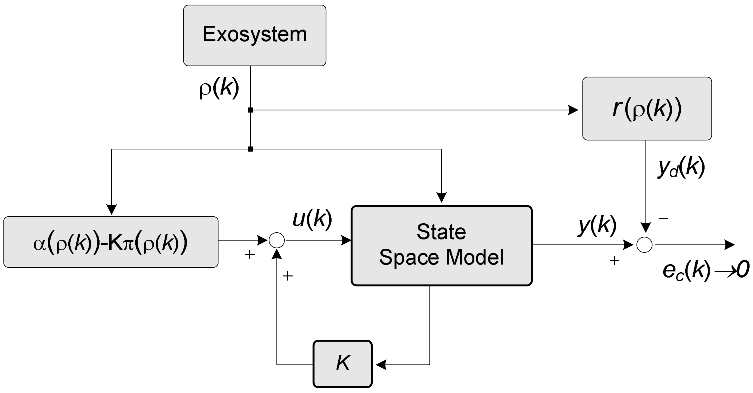


***Figure 5**: Control structure.*

$K\in\Re^{nu\times n}$ is a matrix that places the eigenvalues of the first approximation of the state space model $\left(x(k+1)=\underline{A}x(k)+\underline{B}u(k)\right)$ in desired locations, assuming the pair $\left(\underline{A},\underline{B}\right)$ is controllable. Except in very few cases, it is difficult to derive an analytical solution to the mappings $x(k)=\pi\left(\rho(k)\right)$ and $u(k)=\alpha\left(\rho(k)\right)$ that solve the regulation equations, since it requires solving a set of complex nonlinear difference equations (48). One possibility to overcome this drawback is to solve approximately the regulation equations. Castillo-Toledo [61] have presented and derived conditions for the existence of an approximate solution to the discrete-time case based on a polynomial expansion. Based on a Taylor series expansion, Huang and Rugh [55] have proposed an approximation method for the continuous case. The same authors have presented an alternative approximation (Huang and Rugh, [56]) using a type of RNN, analogous to a cellular network. With a correct choice of parameters, the RNN is able to solve the regulation equations, in the least square sense, by means of a gradient descent minimization. Zhang and Wang [57] propose a neural based control system for continuous systems. By using a power series approximation, a recurrent neural network is developed for online computation of the output regulation feedback gain.

3.4.3 Output Regulation Based on Affine Neural Networks

Based on the particular RNN nonlinearities (hyperbolic tangent), an iterative method that efficiently solves the regulation equations can be derived. The algorithm can be regarded as of gradient type, such that the adaptation gain is adjusted each time in order to guarantee the stability of the iterative procedure. The problem is driven to an eigenvalue assignment, being the convergence guaranteed once the appropriate eigenvalues are selected. When applied to the recurrent neural model (1), the goal is to determine a control law in the form

$$u(k) = \gamma\left(x_n(k), \rho(k)\right) \tag{50}$$

such that the closed loop system is stable and simultaneously condition (51) on neural control error $e_n(k) \in \Re^{ny}$ is held.

$$\lim_{k\to\infty} e_n(k) = y_n(k) - y_d(k) = 0 \tag{51}$$

The application of the zero output constrained dynamics algorithm to the present recurrent neural model leads to the following set of *m* equations $(m = n + n_u)$,

$$\begin{aligned} &\pi\left(\rho(k)\right) = A\pi\left(\rho(k)\right) + B\alpha\left(\rho(k)\right) + D\sigma\left(\pi\left(\rho(k)\right)\right) \\ &0 = C\pi\left(\rho(k)\right) - r\left(\rho(k)\right) \end{aligned} \tag{52}$$

which can be solved through the evaluation of mappings, $x(k) = \pi\left(\rho(k)\right) \in \Re^n$ and $u(k) = \alpha\left(\rho(k)\right) \in \Re^{nu}$. For clarity, it is assumed the following simplified notation $\pi(\rho) \equiv \pi\left(\rho(k)\right)$ and $\alpha(\rho) \equiv \alpha\left(\rho(k)\right)$. It is considered a square system $\left(n_y = n_u\right)$ and a constant reference problem, as well as the existence of a solution $\pi(\rho)^*$ and $\alpha(\rho)^*$ verifying (53).

$$\begin{aligned} 0 &= A\ \pi\left(\rho\right)^* + B\alpha\left(\rho\right)^* + D\sigma\left(\pi\left(\rho\right)^*\right) - \pi\left(\rho\right)^* \\ 0 &= C\ \pi\left(\rho\right)^* - r\left(\rho\right) \end{aligned} \tag{53}$$

Let the vector $\Gamma\left(\rho(k)\right) \in \Re^m$ be comprised of two mappings (54).

$$\Gamma\left(\rho(k)\right) = \begin{bmatrix} \pi\left(\rho(k)\right) \\ \alpha\left(\rho(k)\right) \end{bmatrix} \tag{54}$$

Given a solution $\pi(\rho)^i$ and $\alpha(\rho)^i$ at iteration *i*, one obtains the error $E_1^i \in \Re^n$ and $E_2^i \in \Re^{nu}$, defined as

$$\begin{aligned} E_1^i(k) &= \left[A\pi\left(\rho\right)^* + B\alpha\left(\rho\right)^* + D\sigma\left(\pi\left(\rho\right)^*\right) - \pi\left(\rho\right)^*\right] - \left[A\pi\left(\rho\right)^i + B\alpha\left(\rho\right)^i + D\sigma\left(\pi\left(\rho\right)^i\right) - \pi\left(\rho\right)^i\right] \\ E_2^i(k) &= \left[C\pi\left(\rho\right)^* - r\left(\rho\right)\ \right] - \left[C\pi\left(\rho\right)^i - r\left(\rho\right)\ \right] \end{aligned} \tag{55}$$

Let the solution at iteration $i+1$ given as:

$$\Gamma\left(\rho\right)^{i+1} = \begin{bmatrix} \pi\left(\rho\right)^{i+1} \\ \alpha\left(\rho\right)^{i+1} \end{bmatrix} = \begin{bmatrix} \pi\left(\rho\right)^{i} \\ \alpha\left(\rho\right)^{i} \end{bmatrix} + \begin{bmatrix} \Delta\pi\left(\rho\right)^{i} \\ \Delta\alpha\left(\rho\right)^{i} \end{bmatrix} \tag{56}$$

From (55) the error difference between two iterations $i+1$ and *i*, can be written as.

$$\begin{aligned} E_1^{i+1}(k) - E_1^i(k) &= -A\Delta\pi\left(\rho\right)^i - B\Delta\alpha\left(\rho\right)^i - D\left[\sigma\left(\pi\left(\rho\right)^i + \Delta\pi\left(\rho\right)^i\right) - \sigma\left(\pi\left(\rho\right)^i\right)\right] + \Delta\pi\left(\rho\right)^i \\ E_2^{i+1}(k) - E_2^i(k) &= -C\Delta\pi\left(\rho\right)^i \end{aligned} \tag{57}$$

Since the nonlinearity (hyperbolic tangent) is Lipschitz, one can write (58)

$$\left|\sigma\left(\pi\left(\rho\right) + \Delta\pi\left(\rho\right)\right)\right| \le \left|\sigma\left(\pi\left(\rho\right)\right)\right| + \left|\Delta\pi\left(\rho\right)\right| \tag{58}$$

or, approximately, (59),

$$\sigma\left(\pi\left(\rho\right)+\Delta\pi\left(\rho\right)\right)\approx\sigma\left(\pi\left(\rho\right)\right)+G\Delta\pi\left(\rho\right) \tag{59}$$

being $G\in\Re^{n\times n}$ a diagonal matrix defined by (60), where $\sigma'(\cdot)$ is the derivative of the hyperbolic tangent function.

$$G=diag\left\{\sigma'(\pi(\rho))\right\} \tag{60}$$

Equation (55) can be written as (61),

$$E^{i+1}(k)-E^{i}(k)=-\vartheta\ \Delta\Gamma\left(\rho\right)^{i} \tag{61}$$

where $E^{i}\in\Re^{m}$ is the error corresponding to mappings $\pi(\rho)^{i}$ and $\alpha(\rho)^{i}$ at each iteration *i*. The matrix $\vartheta\in\Re^{m\times m}$ is given by (62),

$$\vartheta=-\begin{bmatrix}-A+DG-I_n & B\\ C & 0\end{bmatrix} \tag{62}$$

where $I_n\in\Re^{n\times n}$ is an identity matrix. Following this approach the problem can be stated as an eigenvalue assignment. Actually, if the increment $\Delta\Gamma(\rho)^{i}$ is given by (63)

$$\Delta\Gamma(\rho)^{i}=F\ E^{i}(k) \tag{63}$$

one obtains,

$$E^{i+1}(k)=\left[I_m-F\vartheta\right]E^{i}(k)=\Lambda E^{i}(k) \tag{64}$$

where $I_m\in\Re^{m\times m}$ and $F\in\Re^{m\times m}$. Thus, it is possible to ensure the error convergence if matrix F is adequately chosen such that $\Lambda\in\Re^{m\times m}$ is Hurwitz.

4. Application to the Solar Power Plant

The strategy presented in this book chapter combines the potentialities of a affine recurrent neural network for approximation purposes along with the output regulation theory for nonlinear control design.

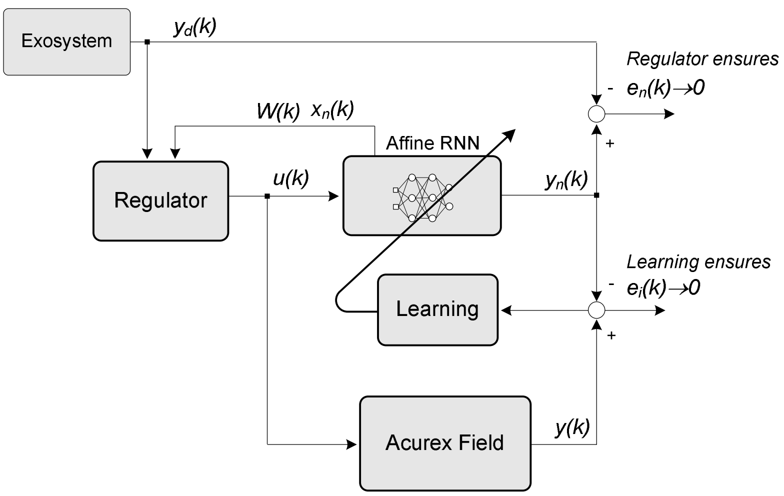


Figure 6: *Proposed control structure.*

However, since the exact knowledge of the parameters is not viable in practice the design of the model based output regulator cannot assure a zero tracking error. To address this problem an adaptive approach is applied with online adjusting of the neural network's weights. A schematic diagram of the proposed adaptive control structure is shown in Figure 6.

4.1 Adaptive Nonlinear Control

Most of the control stabilization results rely on the *Lyapunov* theory to develop a stable control law. A distinct approach consists in the output regulation theory, described in the last section. When applied to the affine RNN (1) the output regulator design ensures (under some constraints) the asymptotic convergence of the neural network tracking error:

$$\lim_{k \to \infty} e_n(k) = y_n(k) - y_d(k) = 0 \tag{65}$$

thus guaranteeing the stability and convergence of the closed loop system. However, the main goal is to ensure that the tracking error $e_c(k)$, (42), converges to zero. Assuming the separation principle, the system tracking error can be written as a function of the identification and control errors,

$$\begin{gathered} e_c(k) = y(k) - y_d(k) \\ e_c(k) = y(k) - y_n(k) + y_n(k) - y_d(k) \\ e_c(k) = e_i(k) - e_n(k) \end{gathered} \tag{66}$$

On the other hand, based on the identification error the learning law is used for updating weights and states, guaranteeing, by means of the Lyapunov theory, the convergence for the error $e_i(k)$:

$$\lim_{k \to \infty} e_i(k) = y(k) - y_n(k) = 0 \tag{67}$$

Thus, $e_c(k)$ will converge provided that the regulator ensures the $e_n(k)$ control error convergence, and the updating algorithm ensures the identification error convergence, $e_i(k)$. Even in the presence of system variations or unmodelled dynamics, the proposed strategy still ensures that the system tracking error will convergence to zero.

4.2 Acurex Field Environment

The experiments were carried out on the Acurex Solar Collectors Field. The proposed control was implemented in C code and operates over a software developed at PSA (López, 1996) also in C code. The effectiveness of the developed approach was first tested using a nonlinear distributed parameter model of the Acurex field, developed at the University of Sevilla (Berenguel et al [58]), (Camacho et al, [59]). The sampling time was chosen as 15 seconds and the output temperature (T_{out}) was considered as the maximum temperature of all the loops (another usual strategy is to assume the average temperature).
Additionally, a feedforward compensation term (Figure 7) obtained using the static behavior of the plant, was introduced to compensate the effects of the radiation and the difference between the desired output temperature and the inlet temperature, [58].

$$Q_{in} = \frac{0.7869 I_{rr} - 0.485\,(\tilde{T}_{ref} - 151.5) - 80.7}{(\tilde{T}_{ref} - T_{in})} \tag{68}$$

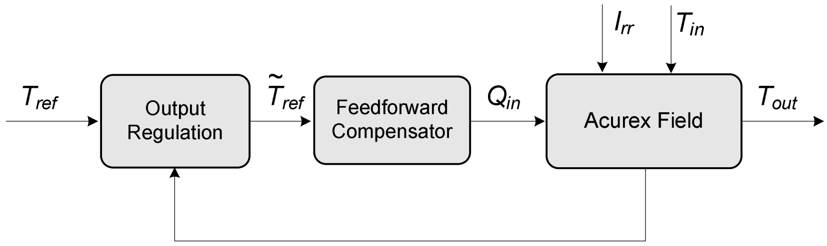


***Figure 7**: Schematic diagram of control scheme the Acurex field.*

5. Experimental Results

5.1 Model Parameterization

The DSC's dynamics is approximated by using the proposed affine state-space neural network structure (1). In order to come up a with a particular parameterization for this topology the Acurex field was prior subjected to a series of experiments so as to collect informative data sets to be used during the subsequent training stage. Figure 8 presents one of those data sets.

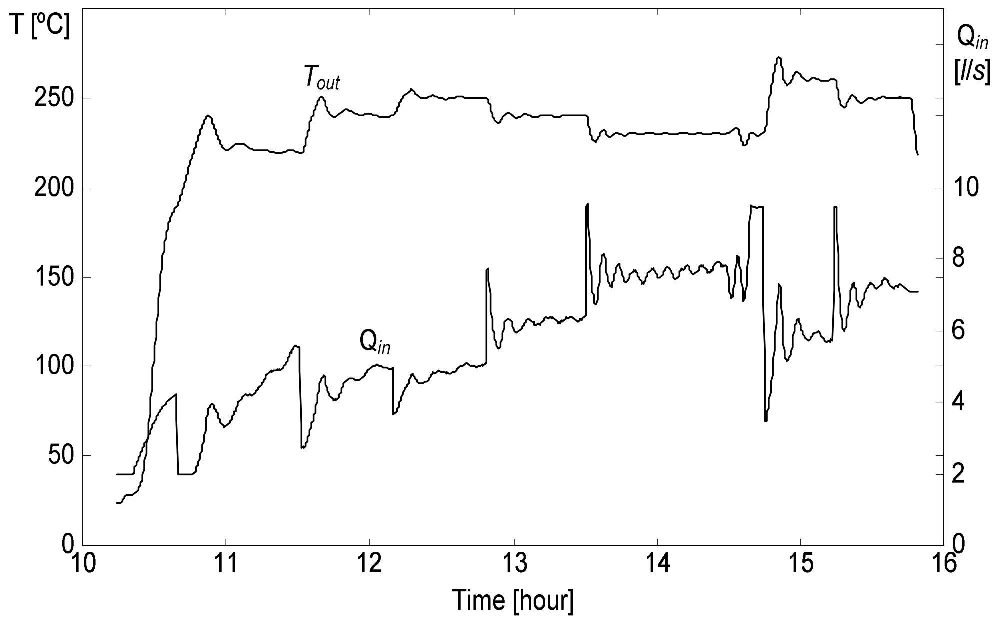


***Figure 8**: Data set collected on July 23.*

By applying the proposed methodology to estimate the number of hidden-layer neurons, based on the SVD of $\tilde{\Pi}_i$, and considering the number of block rows as 15 the following singular values were obtained:

$$\sigma = \begin{pmatrix} 1279.9 \\ 27.79 \\ 8.5948 \\ 1.0603 \\ 0.6615 \\ 0.4998 \\ 0.3634 \\ 0.3539 \\ 0.3335 \\ 0.3026 \\ 0.2701 \\ 0.2568 \\ 0.1156 \\ 0.0987 \\ 0.0042 \end{pmatrix}$$

As can be observed from the singular values vector, the cardinality of the dominant ones is at most 3 and consequently, according to the rationale behind the proposed approach, the affine state space neural network order is chosen as 3. In Figure 9 it is presented the prediction performance of this particular topology trained using the Levenberg-Marquardt algorithm.

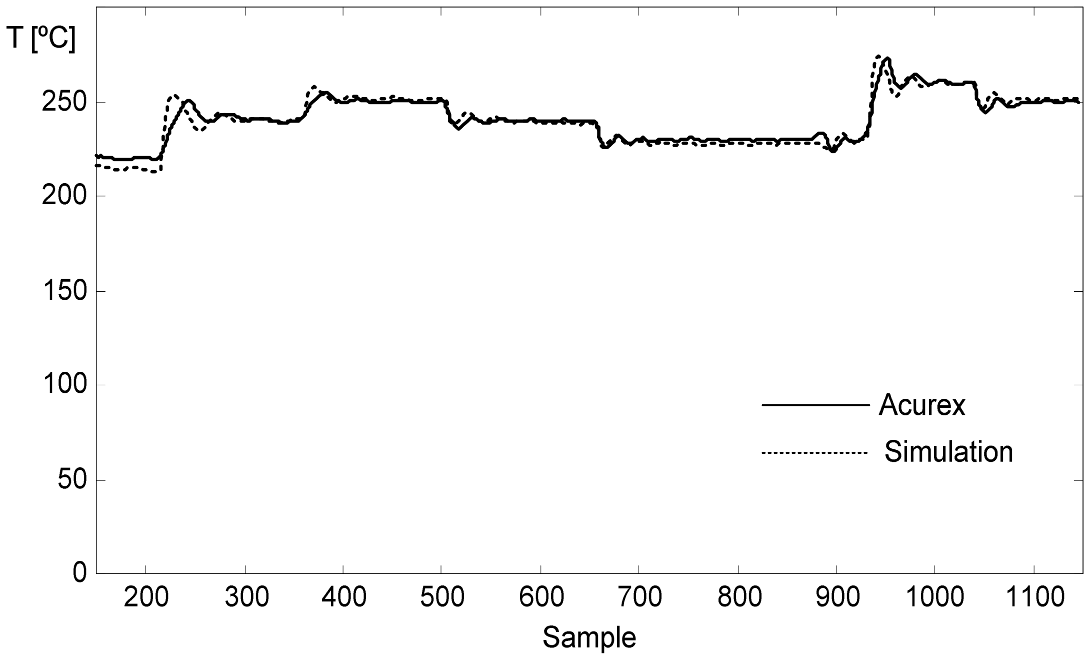
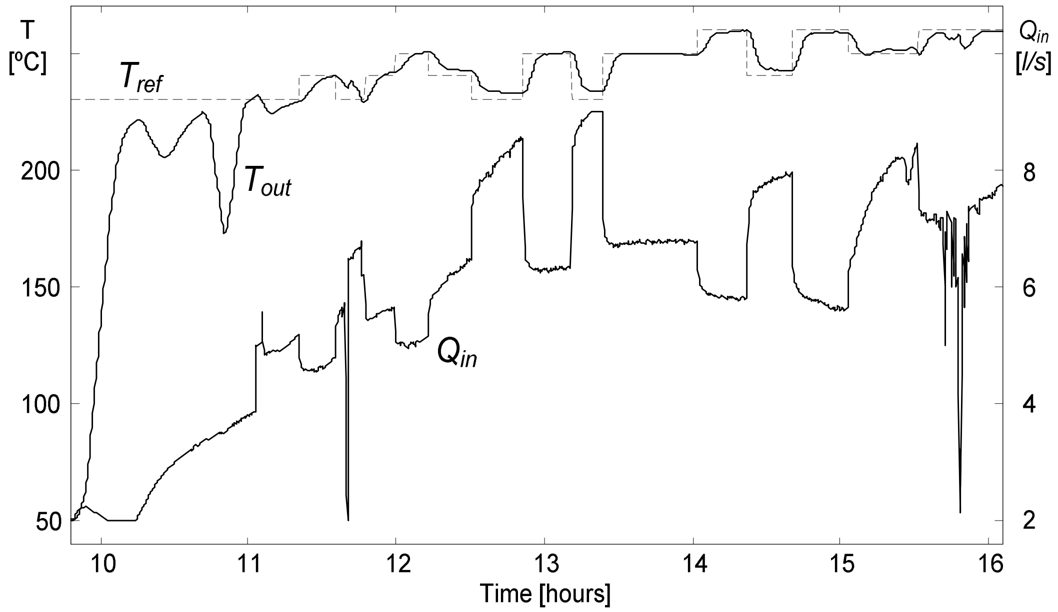


Figure 9: *Measured and predicted outlet oil temperature.*

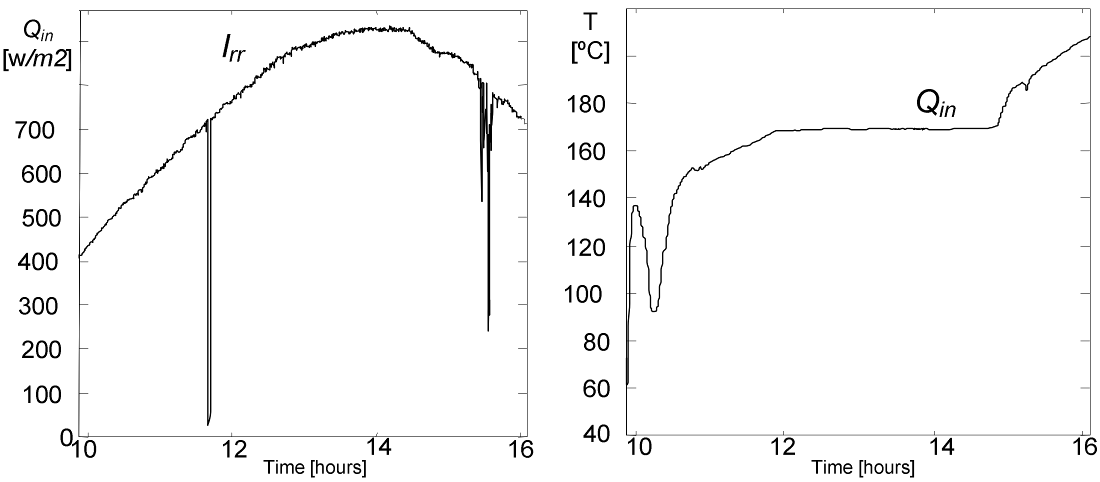
From the comparative results, in terms of prediction capabilities for the outlet oil temperature, it is notorious that the affine state space neural network is able of capturing the nonlinear system dynamics embedded in the training data set.

5.2 Control

The present experiment was carried out to assess the performance of the control system on the real plant considering different reference temperatures. Fortunately, the effect of strong disturbances caused by large passing clouds, which produce drastic changes in the direct solar radiation level was possible to test, as well as the effect of inlet oil temperature variation. As can bee seen in Figure 10, the closed loop performance is quite acceptable. The response presents almost no oscillations neither overshoots and after an initial transient period the outlet oil temperature stabilizes close to the reference.



(a) Reference, output temperature and pump flow rate.



(b) Solar radiation and inlet oil temperature.

Figure 10: *Experimental results obtained.*

Although, it should be expected a zero steady state error, the actual steady state off-set error is justified by the lower gain characteristics of the online learning. Since it was not possible to adjust during the operation both learning and controller parameters their choice were not the most favorable. In Figure 10 the behavior of the closed loop system when intermittent clouds occur (11h50m and 15h40m) can also be analyzed. They lead to changes in solar radiation that disturb the outlet oil temperature level during operation. As observed the control action is very acceptable in this case. The disturbance rejection capabilities of the controller are also acceptable, resulting in a change in the inlet oil temperature, carried out at instant 15h00m.

From several simulations, tested by using the nonlinear distributed parameter model of the Acurex field, ([58], [59]), together with the experiments, it can be inferred that the neural output regulation strategy performs according to its design. By online adjusting the neural parameters it is possible to reduce gradually the model plant mismatch contributing to the convergence of the tracking error to zero. Moreover, it provides a control law such that the closed loop system is stable.

5.3 Discussion

It is known that a fixed parameter PID controller can provided acceptable results in the case of linear systems. However, when dealing with non-linear processes, the overall closed-loop performance deteriorates for different operating conditions and disturbances. The proposed control scheme is able to inherently deal with these non-linearities, providing substantial improvements in terms of closed loop performance. Actually, apart from closed loop robustness, the output presents almost no overshoot and a settling time of about 15 minutes. This means that the current non-linear strategy is able to capture the underlying non-linear system dynamics, providing remarkable results. Moreover, since the model parameters are adjusted at each sampling time, based on the specific plant conditions, it is possible to cope with the model plant mismatches by adjusting, in real time, the model parameters. Regarding the controller scheme, the output regulation theory was considered given the ability to design a stable closed loop control system, as well as to ensure the convergence of the control error.

6. Conclusions

An indirect adaptive nonlinear control scheme combining an affine recurrent neural network and the output regulation theory was implemented in real-time and applied on a distributed solar collector field.

In order to manage the network complexity it was used a subspace based technique where the estimated model order can be regarded as a majorant for the number of neurons to be inserted in hidden layer.

To cope with the inaccuracy of offline estimated neural parameters, and possible changing dynamics, an adaptive strategy was applied. The methodology was based on an online identification, ensuring stability and convergence properties, by means of the *Lyapunov* theory. Thus, the neural model can adaptively learn the system dynamics and the regulator law adjusts the control action in order to guarantee an asymptotic control error convergence.

Experimental results confirm those obtained in simulation and show that the system has robustness with respect to changes in solar radiation, inlet oil temperature and operating conditions. This experimental study has shown that neural networks are an important methodology for many industrial control applications. The simplicity and reliability of neuro-control presents high potential for the development of efficient and intelligent control systems.

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