

# Routing Policy Choice Set Generation in Stochastic Time-Dependent Networks: Case Studies for Stockholm and Singapore

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## 1 **Abstract**

2 Transportation systems are inherently uncertain due to the occurrence of random disruptions; meanwhile, real-time  
3 traveler information offers the potential to help travelers make better route choices under such disruptions. This paper  
4 presents the first revealed preference (RP) study of routing policy choice where travelers opt for routing policies  
5 instead of fixed paths. A routing policy is defined as a decision rule applied at each link that maps possible realized  
6 traffic conditions to decisions on the link to take next. It represents a traveler's ability to look ahead in order to  
7 incorporate real-time information not yet available at the time of decision. An efficient algorithm to find the optimal  
8 routing policy (ORP) in large-scale networks is presented, as the algorithm is a building block of any routing policy  
9 choice set generation method. Two case studies are conducted in Stockholm, Sweden and Singapore, respectively.  
10 Data for the underlying stochastic time-dependent network are generated from taxi Global Positioning System (GPS)  
11 traces through the methods of map-matching and non-parametric link travel time estimation. The routing policy choice  
12 sets are then generated by link elimination and simulation, in which the ORP algorithm is repetitively executed. The  
13 generated choice sets are first evaluated based on whether or not they include the observed GPS traces on a specific  
14 day, which is defined as coverage. They are then evaluated on the basis of adaptiveness, defined as the capability  
15 of a routing policy to be realized as different paths over different days. It is shown that using a combination of link  
16 elimination and simulation methods yield satisfactory coverage. The comparison to a path choice set benchmark  
17 suggests that a routing policy choice set could potentially provide better coverage and capture the adaptive nature of  
18 route choice. The routing policy choice set generation enables the development of a discrete choice model of routing  
19 policy choice, which will be explored in the second stage of the study.

# 1 Introduction and Background

2 Transportation systems are frequently subject to random disruptions such as incidents and bad weather, resulting in  
3 variable and unpredictable traffic conditions. Meanwhile, real-time information of realized traffic conditions could  
4 potentially help reduce the uncertainty and help travelers make more informed decisions. As a result, Advanced  
5 Traveler Information Systems (ATIS) have been identified as a potential strategy towards reducing congestion and  
6 improving system reliability. A crucial component in designing and evaluating ATIS is understanding route choice  
7 behavior of travelers in response to a wide range of traveler information situations in a network with dynamic and  
8 random traffic conditions.

9 A traveler makes decisions based on her knowledge of the available alternatives and their attributes. This  
10 knowledge is periodically updated by both personal experience and exogenous information, and as a result the deci-  
11 sions might be revisited and revised. In other words, a traveler "adapts" to the decision environment. The time scale at  
12 which route choice adaption happens can be broadly divided into two types: day-to-day and with-in day. In a day-to-  
13 day context, a traveler's route choice today might be different from yesterday due to information collected yesterday  
14 during the trip. In a within-day context, route choice could be revised en route, e.g., taking a detour upon receiving  
15 information on an accident on the bridge along the original route. This paper focuses on within-day adaptive route  
16 choice, where the real-time information reflects travel conditions at or close to the decision time. For a recent review  
17 of empirical day-to-day route choice studies, please refer to (1).

18 A large body of research on route choice in response to real-time information focuses on binary route switch-  
19 ings in real life, e.g., (2), (3), (4), (5), or more advanced hypothetical ATIS in stated preference (SP) surveys, in which  
20 we directly ask travelers about their preferences for route choice in hypothetical situations, e.g., (6), (7), (8), (9), (10),  
21 (11). In all of these studies, travelers are assumed to respond to real-time information on the spot, and the complete  
22 decision process is a series of path choices, each of which is based on updated traffic conditions revealed by real-time  
23 information at the time of decision. The implicit assumption is that a traveler is myopic and cannot look ahead for  
24 future information. Such behavior is defined as adaptive path choice.

25 Some researchers, e.g. (12), (13), (14), study the response before the information is received for travelers with  
26 looking-ahead abilities. They find that a traveler does not need to commit to a particular route, but instead can decide  
27 later at a switching point based on then revealed traffic conditions, and choose the route with a shorter travel time for  
28 the remaining trip. The option value of downstream real-time information thus could potentially make a collection of  
29 alternatives that share a common starting link more attractive than other links out of the same decision node. Therefore,  
30 the travelers respond to the information upstream of the actual point where it is received. Such travelers are said to  
31 take routing policies, which loosely speaking, are decision rules applied at each link that map possible realized traffic  
32 conditions to decisions on the link to take next. Empirical studies of the routing policy choice to this date have only  
33 been carried out with SP data.

34 This research contributes to the state of art by conducting the first revealed preference (RP) study of routing  
35 policy choice using Global Positioning System (GPS) data. RP studies of adaptive route choice (adaptive path and  
36 routing policy choice) in real life networks impose challenges in data collection. Three major types of data are  
37 needed: travelers' chosen routes, network conditions, and travelers' information access. The first two types of data  
38 are increasingly more available due to the advent of GPS technologies. For example, (15) compared the planned and  
39 actually taken routes (observed by GPS) of travelers and found that 20% of surveyed travelers switch routes for various  
40 reasons (one of them being ATIS). There have been a large number of GPS data collection efforts throughout the world,  
41 e.g., (16), (17) (18), especially with the ever increasing popularity of GPS-enabled smart phones. The third type of  
42 data, however, is not available from passive GPS traces, and must be collected through surveys or other monitoring  
43 devices such as video cameras. This research uses taxi GPS traces from two urban areas: Stockholm and Singapore to  
44 generate chosen routes and link travel times for the network.

45 Another challenge of RP routing policy choice studies is computational. The underlying network for adaptive  
46 route choice is conceivably more complicated than that for a fixed path choice model, as travel times are usually  
47 represented as time-dependent random variables to support modeling the dynamic and adaptive nature of the behavior.  
48 The alternative in the choice set is a routing policy, and the choice set generation thus requires repetitive executions  
49 of an optimal routing policy (ORP) algorithm, which in general is much more time consuming than shortest path  
50 algorithms in deterministic networks. There have been a large number of algorithmic studies which generate optimal  
51 routing decisions depending on traffic conditions revealed by real-time information in a stochastic network, e.g., (19),  
52 (20), (21), (22). An efficient ORP algorithm applicable in large-scale real life networks is developed in (23) and is  
53 used in this study.

1 This paper presents the first step in the RP study of routing policy choice, i.e., routing policy choice set  
 2 generation. For real-life networks, there may exist a large number of routes/routing policies for an origin-destination  
 3 (OD) pair. Some studies avoid choice set generation in the context of route choice model estimation. (24) presents a  
 4 new paradigm for choice set generation by assuming that the choice sets contain all paths connecting each OD pair,  
 5 and a sampling approach was proposed to generate subsets of paths suitable for model estimation. Despite the fact that  
 6 this approach avoids the bias in model estimation, the application is not feasible when calculating the probability of  
 7 routes in the universal choice sets. (25) proposes a dynamic discrete choice approach for consistently estimating route  
 8 choice model parameters based on path observations through repeated link choices. The approach does not require  
 9 choice set generation or sampling. It currently only applies to non-adaptive path choices.

10 Unless the universal set is treated as the choice set as in (24) and (25), a subset of reasonable alternatives  
 11 need to be generated. There are deterministic and stochastic algorithms in path choice set generation (26). Commonly  
 12 used deterministic methods include link elimination (27), (28), (18), link penalty (29), labeling (30), constrained  
 13 k-shortest paths (31), and branch-and-bound ((32) for public transport networks, (33) for multi-modal networks and  
 14 (34) for private transport networks). Stochastic methods include simulation (35), (36), and doubly stochastic choice set  
 15 generation (37). (38) provides a comprehensive comparison of a large number of path choice set generation algorithms  
 16 using a data set from Boston. (39) suggests that transport modelers should implement stochastic path generation  
 17 techniques with average variance of its distribution parameters, and correction for unequal sampling probabilities of  
 18 the alternative routes in order to obtain satisfactory results for coverage of "postulate chosen route", and reproduction  
 19 of "true model estimates". We investigate a number of routing policy choice set generation algorithms which are  
 20 generalizations of the link elimination and simulation methods for path choice set generation.

21 This paper has been structured as follows. Section 2 begins by introducing the modeling framework and  
 22 choice set generation methods in which an ORP algorithm is needed as a building block. Section 3 presents a network  
 23 data processing methodology, which may be used to process real-life network data for choice set generation. In Section  
 24 4 these methodologies are implemented in two real-life case studies with one in Stockholm, Sweden and the other in  
 25 Singapore. A benchmark analysis is also carried out on path choice set generation for comparison. In Section 5 the  
 26 conclusions are made and future directions are introduced.

## 27 **2 MODELING FRAMEWORK AND CHOICE SET GENERATION**

### 28 **2.1 Network, Information, Route Choice Behavior**

29 The network is modeled as stochastic time-dependent (STD), in which link travel times are jointly distributed time-  
 30 dependent random variables. The STD network is denoted as  $G = (N, A, T, P)$ , where  $N$  is the set of nodes,  $A$  the set  
 31 of links with  $|A| = m$ ,  $T$  the set of time periods  $\{0, 1, \dots, K - 1\}$ , and  $P$  the probabilistic representation of link travel  
 32 times. Beyond time period  $K - 1$ , travel times are static and deterministic.  $T(i, j, k, t)$  is the deterministic turning  
 33 movement penalty from link  $(i, j)$  to link  $(j, k)$  at time  $t$ .

34 Belonging to the link travel time representation, a "support point" is defined as a distinctive value that a  
 35 discrete random variable can take, or a distinctive vector of values that a discrete random vector can take depending  
 36 on the context. Thus a probability mass function (PMF) of a random variable (or vector) is a combination of support  
 37 points and the associated probabilities. A joint probability distribution of all link travel time random variables is used:  
 38  $P = \{v_1, v_2, \dots, v_R\}$ , where  $v_i$  is a vector with a dimension  $K \times m$ ,  $i = 1, 2, \dots, R$ , and  $R$  is the number of support  
 39 points. The  $r^{th}$  support point has a probability  $p_r$ , and  $\sum_{r=1}^R p_r = 1$ . When link travel time observations from multiple  
 40 days are available, a support point can be viewed as a day,  $R$  is the number of days, and  $p_r = 1/R, \forall r$ .

41 Real-time information is assumed to include realized travel times of certain links at certain time periods.  
 42 For example, perfect online information (POI) includes realized travel times on all links up to the current time, while  
 43 global pre-trip information includes realized travel times of all links up to the departure time. See (22) for discussions  
 44 on a number of real-time information access. The passive GPS traces of taxi drivers used in this study cannot tell  
 45 us what real-time information the drivers have. We assume POI, since taxi drivers are in general highly sensitive to  
 46 traffic conditions and stay informed at all times. For example, the land transport authority of Singapore uses real-time  
 47 taxi GPS data (from the same taxi operator as in the case study) to predict travel times and traffic conditions. This  
 48 information is shown on road-side electronic boards, offered on Google maps, and sent to taxi mobile data terminals  
 49 (MDT). The discussion in the remainder of the paper is therefore specific to POI.

50 At a given time period  $t$ , the available real-time information is represented by a joint realization of travel

1 times on all links at time periods  $0, 1, \dots, t$ . The joint realization corresponds to a unique subset of compatible support  
 2 points, defined as an event collection,  $EV$ , which represents the conditional distribution of link travel times given the  
 3 realization of link travel times. As more information becomes available, the size of an event collection decreases or  
 4 remains the same. When an event collection becomes a singleton, the network becomes deterministic.

5 When a traveler is at the start of link  $(i, j)$  at time  $t$  with event collection  $EV$ , he/she makes a decision to  
 6 take the next link  $(j, k)$ . Upon arrival at node  $j$  (end of link  $(i, j)$ ), he/she will be in a different time period due  
 7 to the traversal time on link  $(i, j)$  and the turning penalty  $T(i, j, k, t)$ . He/she will also have a potentially different  
 8 event collection  $EV'$ , which accounts for realized link travel times between  $t$  and the arrival time at node  $j$ . He/she  
 9 continues the routing decision process based on dynamically involved event collections. Define  $x$  as a state with three  
 10 elements: link  $(i, j)$ , time  $t$  and event collection  $EV$ . A routing policy  $\mu$  is therefore defined as a mapping from all  
 11 possible states to the decision of the link to take next,  $\mu : x \mapsto (j, k)$ .

12 A routing policy can capture traveler's looking-ahead capability in that the decision at state  $x$  depends on  
 13 the evaluation of all possible future states throughout the remainder of the trip by following each outgoing link.  
 14 Specifically, the fact that more information will be available in the future is represented by the series of  $EV'$  that could  
 15 be encountered. A routing policy is realized as a path on a given support point (day), and the realized path topologies  
 16 potentially vary from day to day due to the randomness of travel times and information.

## 17 2.2 Optimal Routing Policy

### 18 2.2.1 The Optimality Condition

19 An optimal routing policy (ORP) algorithm is a building block of any routing policy choice set generation method. In  
 20 an STD network, the ORP is a routing policy that moves a traveler on a network from the origin to the destination at  
 21 the least cost, which can be the least expected travel time in a simple case. Define  $e_{\mu}(x)$  as the expected travel time  
 22 to the destination node  $d$  with the routing policy  $\mu$  and the initial state  $x$ . Let  $\tau_{jk,t}^{EV}$  be the travel time on link  $(j, k)$   
 23 at time  $t$  given event collection  $EV$ , which is a single value due to the assumption of POI (all link travel times at  
 24 time  $t$  are known).  $A(j)$  is the set of downstream nodes of node  $j$ , and  $B(j)$  is the set of upstream nodes of node  $j$ .  
 25  $Pr(EV'|EV)$  is the probability of  $EV'$  conditional on  $EV$ .  $EV(t)$  is the set of all possible event collections at time  
 26  $t$ .

27  $e_{\mu_{\star}}(x)$  and  $\mu_{\star}$  are optimal solutions if and only if they satisfy the following equations:

$$e_{\mu_{\star}}(i, j, t, EV) = \min_{k \in A(j)} \left\{ \sum_{EV' \in EV(t + \tau_{ij,t}^{EV} + T(i, j, k, t))} e_{\mu_{\star}}(j, k, t + \tau_{ij,t}^{EV} + T(i, j, k, t), EV') \right. \\ \left. \times Pr(EV'|EV) + T(i, j, k, t) + \tau_{ij,t}^{EV} \right\} \quad (1)$$

$$\mu_{\star}(i, j, t, EV) = \operatorname{argmin}_{k \in A(j)} \left\{ \sum_{EV' \in EV(t + \tau_{ij,t}^{EV} + T(i, j, k, t))} e_{\mu_{\star}}(j, k, t + \tau_{ij,t}^{EV} + T(i, j, k, t), EV') \right. \\ \left. \times Pr(EV'|EV) + T(i, j, k, t) + \tau_{ij,t}^{EV} \right\} \quad (2)$$

28 with boundary conditions:  $e_{\mu_{\star}}(d', d, t, EV) = \tau_{d',d,t}^{EV}, \forall t \in T, \forall d' \in B(d), \forall EV$ , and  $e_{\mu_{\star}}(i, j, t, EV) = e_{\mu_{\star}}(i, j, K -$   
 29  $1, EV), \forall (i, j) \in A, \forall t > K - 1, \forall EV$ .

### 30 2.2.2 Algorithm LC-CDPI

31 Based on the optimality condition, Algorithm LC-CDPI (Label-Correcting Complete Dependency Perfect Online In-  
 32 formation) has been previously designed in (23) and is applied in this research. This algorithm determines the least  
 33 expected travel times from all nodes at all departure times with all possible current-information to a certain destination  
 34 node  $d$ . Algorithm LC-CDPI implements the following three major changes to the original label-setting Algorithm  
 35 DOT-SPI (21) to make it applicable to large networks with improvements in realistic modeling features, memory, and  
 36 runtime.

**Piece-wise Linear Travel Time Representation** The dynamic link travel times are represented by piece-wise linear functions instead of discrete values. In a typical application of the ORP problem, the study period (e.g., 6-9 am) is divided into smaller time intervals each having a duration of 5 to 15 minutes. The discrete joint link travel time distribution in Algorithm DOT-SPI is represented by a three-dimensional matrix with the dimension of "number of time intervals  $\times$  number of support points  $\times$  number of links." For real networks with hundreds of thousands of links, it is nearly impossible to store the distribution in the computer memory for a reasonably long study period. A piece-wise linear function, on the other hand, can store only the breaking points. Between the two breaking points, the travel time is derived by interpolation. Since the breaking points are generally sparsely distributed over time, the number of breaking points is much smaller than the number of time intervals. For example, a study period of 3 hours with 5-minute time intervals has 60 time intervals, and if the piece-wise linear function is defined by only 10 points, which in general is adequate to describe the profile of travel time over the 3-hour horizon, then the memory required is reduced by 6 times.

**Label-correcting** Algorithm DOT-SPI is a label-setting algorithm that goes through all time intervals in decreasing order of time, as the name DOT (decreasing order of time) suggests. It requires link travel times to be positive integers in units of time interval length such that the time-expanded network is acyclic in the time dimension and label setting can be applied. As a result, the time interval length needs to be smaller than the shortest link travel time in the network. This constraint usually results in a time interval length in seconds, and therefore a very large number of time intervals. This large number of intervals leads to very long runtime (in addition to the excessive memory usage, which could be resolved by the previously introduced piece-wise linear function). We have designed a label-correcting algorithm that only requires the travel times to be non-negative, and thus the time interval length can be greater than link travel times. As such, the number of time intervals can be significantly reduced, resulting in the reduction of runtime.

**Turn-based** The behaviors of waiting at nodes, making U-turns, and looping are infrequent in a real-life network since travelers want to avoid loss in travel time, but such problems are observed when applying Algorithm DOT-SPI in several real-life networks. Algorithm DOT-SPI cannot avoid such problems because it is node-based and turning penalties cannot be added. The improved Algorithm LC-CDPI is a link-based algorithm so turning penalties can be applied to make waiting, U-turns, and looping less desirable.

### 2.2.3 Complexity Analysis

Incorporating the changes stated above, Algorithm LC-CDPI consists of two major steps. The first step is to construct event collections, and it takes runtime  $O(mKR \ln R)$  and  $\Omega(mKR)$ . The second step is the main loop of the label correcting procedure, which updates labels at all links, all time periods and for all event collections until the optimality conditions are satisfied, and it takes runtime  $O(m^3K^2R)$ . Notably, the worst case complexity of the algorithm is thus  $O(mKR \ln R + m^3K^2R)$ .

Practically, however, the computational tests in (23) show that the runtime is linear relative to most of the size factors, e.g. number of time periods, number of support points, number of links, while the worst case complexity indicates faster growth. This result suggests the adverse worst case complexity rarely occurs in practice and the algorithm yields good average case performance.

## 2.3 Choice Set Generation

The Algorithm LC-CDPI then contributes to the generation of choice sets. There could be numerous alternative routes/routing policies in a general transportation network for an OD pair, but many of them may be unrealistic by being too circuitous or otherwise unsuitable. Therefore, the objective of the choice set generation is to provide a subset of realistic alternatives considered by a traveler.

For looking-ahead travelers, the alternatives are routing policies instead of simple paths, and thus the choice set generation is different from that in conventional path choice context. Similar to the shortest path algorithm as needed for a path choice set generation, an ORP algorithm (Algorithm LC-CDPI) is used for the routing policy choice set generation as an elementary component of the link elimination and simulation methods. If necessary, other methods as mentioned in Section 1 could potentially be generalized for the routing policy choice set generation. The common aspect of these methods is the repeated calculations of the ORP (or shortest path) in slightly modified versions of the original network.

### 2.3.1 Link Elimination Method

The first method utilized is link elimination, which is similar to that in conventional path choice context. In this method, the ORP is first calculated for an OD pair. Links on the ORP are then removed from the network one at a time, and a new ORP is generated and added to the choice set if not already included. These links are removed at regular link intervals, which is referred to as the elimination interval. Removing consecutive links is likely to generate identical routing policies because these links are close in proximity and similar in functional class, such as consecutive highway links. Thus the introduction of the elimination interval can help reduce redundancy and also runtime. This interval can be decided empirically based on the network parameters such as the estimated actual number of links on the observed paths.

Despite the use of the elimination interval, it is still possible that the newly generated routing policies are similar to the original one. Since only one link is removed each time, the new routing policy generated at each iteration may only differ from the original by a short detour around the eliminated link. Therefore, the simulation method is also utilized to ensure the diversity of the choice set.

### 2.3.2 Simulation Method

In the simulation method, an independent distribution is adopted to generate the cost of every link for each time period and support point. For each independent sample of link travel times, an ORP is generated and added to the choice set if different from any existing alternative. The number of samples is pre-determined based on network parameters and can be adjusted empirically.

## 2.4 Choice Set Evaluation

The generated choice sets are then evaluated to verify that they contain no duplicate routing policies, and have sufficient coverage and adaptiveness. In general, they should not contain routes that no traveler would consider or exclude routes that travelers may choose.

### 2.4.1 Coverage

Sufficient coverage is an indication of a successful choice set. In an ideal condition, a generated route in the choice set for a particular OD pair should match the observed path link by link so that the choice set has included the observed path. In this case, coverage is the percentage of path observations contained in the generated choice set. To be practical, we relax the matching criterion because not all the observed routes will necessarily be generated link by link. To quantify this criterion, overlap is introduced as a percentage of the observed route's deterministic and static travel time shared by the generated route and the observed route. Coverage is in turn redefined as the percentage of path observations for which the algorithm has generated a routing policy whose realization on the given day (a path) meets a particular threshold for overlap.

In the case studies, only GPS traces, not the actual chosen paths are observed. Routing policies are also not observable, and realized paths on the observation day are used. For a given OD pair, the policies in the choice set are realized as particular paths on the day that the GPS observation is recorded. If any one of those paths sequentially contains all the GPS traces, then this OD pair is a match under 100% overlap. If the matching criterion is broadened, e.g., 80% overlap threshold, then other link parameters such as deterministic and static link travel times are needed to calculate the coverage.

### 2.4.2 Adaptiveness

Adaptiveness is defined as the fraction of days when a routing policy is realized as different paths. Adaptiveness for a given OD pair is then averaged over all policies in the choice set. Sufficient adaptiveness is a new criterion for evaluating routing policy choice sets, which is not used in evaluating path choice sets. This criterion is introduced to show the advantages of routing policies over paths by requiring the routing policy choice sets to show adaptive routing of travelers. Sufficient adaptiveness requires a routing policy to be realized as different paths on different support points (days), while paths are fixed over days.

### 3 Network Data Processing Methodology

Three data sets are needed for choice set generation and evaluation: network topology, vehicle trajectories, and link travel time distributions. GPS traces are used to generate the latter two data sets. Each trace consists of latitude, longitude, timestamp, status (free/hired), and the unique vehicle ID. The trace generation process is time-based and the gap between traces varies depending on the location. The GPS data is then processed using map-matching methods to produce two sets of output used in the subsequent choice set generation and evaluation: vehicle trajectories and dynamic link travel time distributions.

For hired taxi data, we refer to trips with passenger(s) on board, whose duration is between meter on and meter off. It is assumed that when there are passengers on board, taxi drivers have a clear origin and destination, and have routing goals similar to those of general drivers, whereas non-hired taxis roam the network in order to pick up passengers. It is likely that taxi drivers are more experienced, aggressive, and knowledgeable about the area and take a route that some commuting drivers would not take such as back roads or shortcuts to avoid difficult intersections or traffic areas. Therefore, the developed model will represent behaviors of a subset of the general drivers who are knowledgeable about the network and sensitive to real-time traffic information. The methodology can be applied in the future to model more general drivers' behaviors when data becomes available.

#### 3.1 Network and Map-Matching

The network is represented as a directed graph with links for streets, nodes for intersections, and locations where link attributes change. Each link has a number of attributes including speed limit, functional class and presence of traffic signal. For this application, the network is simplified so that links in series with identical speed limit and functional class attributes are merged, reducing time and memory requirements of subsequent processing. Furthermore, to ensure that there exists a path between every pair of nodes, only the largest strongly connected components of the network are used.

The GPS traces are matched to the road network using a 4-step map-matching method designed for sparse Floating Car Data (FCD), which is data collected from traced vehicles that "float" with the traffic flow (40). The method first finds candidate links in the vicinity of each trace, then connects the candidate links of each pair of traces. The method then creates a candidate graph between a sequence of traces and, finally, finds the most likely path from the candidate graph.

#### 3.2 Vehicle Trajectories

The start and end of each vehicle trajectory are identified through changes in the vehicle status from free to hired and vice versa since only hired taxi trips are used. For each trajectory, the data includes the vehicle ID, origin, destination, date, start time, duration of the trip, number of GPS traces, the sequence of links where the vehicle sent the traces (determined through the map-matching), and the corresponding sequence of link entry times.

#### 3.3 Link Travel Time Distribution

A non-parametric method (41) is used to compute the link travel time distribution per time interval using the map-matched GPS data. For each road segment between a pair of traces, the observed travel time (i.e., the difference between the time stamps) is decomposed to the traversed links considering their overlapping lengths. The weighted average, where the weight reflects the overlap with both the considered link and other links, over observations from different vehicles within the same time interval is the estimated link travel time. The travel time estimation is performed for each time interval separately for each day in the data set, producing an empirical travel time distribution with the joint support points as days.

With the available data, there are link-day-interval combinations for which the travel time cannot be estimated due to lack of observations. These missing values are filled in through a sequence of inter/extrapolation steps. Furthermore, unreasonably high or low link travel times are removed to produce reliable estimates. Links with many missing values before interpolation are treated as deterministic, and a single mean travel time is estimated across all days.



### 3.4 Data Evaluation

After acquiring and processing the GPS data, an evaluation is conducted regarding factors such as the frequency of GPS traces, the level of congestion, and uncertainties of the network. For instance, GPS traces with a time gap larger than a certain threshold are filtered out to ensure accurate observations of chosen routes. Sub-networks with high levels of congestion and uncertainties are identified and extracted to allow potential adaptive route choices. They are also simplified by merging links with the same attributes and connected by nodes that are not real-life intersections. The runtime and memory are thus reduced by the subnetwork extraction and link/node simplification. For example, in the Singapore case study, before merging there are 71,373 nodes and 80,346 links; after merging and before cutting, there are 19,914 nodes and 28,329 links; after cutting, there are 7,808 nodes and 11,106 links.

## 4 Case Studies

To apply the methodologies to real-life networks, two case studies are conducted with one in Stockholm, Sweden and the other in Singapore. GPS data sets from the two sub-networks are investigated, and the statistics are listed in Table 1.

Statistic	Network	
	Stockholm	Singapore
# of Nodes	3,122	7,808
# of Links	5,845	11,106
# of Stochastic Links	617	554
# of Taxis	1,500	15,000
# of Support Points	56	59
Traces Time Gap	1-2 min	3-4 min
# of OD Pairs Evaluated	997	4684
# of Time Intervals	30	98
Time Interval Length	5 min	5 min in peak hours and 30 min in off-peak hours
Study Time Period	6:30 AM - 9:00 AM	24 Hours
Link Elimination Interval	10	10
# of Simulations	30	30

TABLE 1 : Network Statistics for Choice Set Generation Case Studies

### 4.1 Computational Requirements

The case studies are conducted on ordinary desktop configurations. The methodologies are computationally efficient and can be applied in real-life large networks.

#### 4.1.1 Computer Specifications

For the Stockholm case study, the computer specifications are Intel(R) Core(TM) i5 CPU 650 @ 3.20GHz, 3193 Mhz, 2 Core(s), 4 Logical Processor(2), 8G memory. For the Singapore case study, the computer specifications are Intel(R) Xeon(R) CPU E5-2620 0 @ 2.00GHz, 32G memory.

#### 4.1.2 Memory

For data processing, the required memory depends on the network size. For instance, the map-matching and path inference module requires approximate 1 GB in the case of using the entire Stockholm network. For the Stockholm case study, however, 256 MB is enough since the sub-network size is relatively small. The map-matching module reads the input data (GPS traces) as a stream, and writes the output, which are trajectories, as another stream. This means that memory consumption does not depend on the size of the input data.

1 For choice set generation, the major memory usage is from the storage of link travel times, which requires  
2 space calculated as Number Of Links  $\times$  Number Of Support Points  $\times$  Number Of Breaking Points  $\times$  Storage Unit.  
3 For example, if the travel times are stored as float numbers, the memory used for this storage would be approximate  
4 16.2 MB for the Singapore case study, and 4.6 MB for the Stockholm case study. Therefore, an ordinary desktop  
5 configuration would satisfy the requirements.

### 6 4.1.3 Runtime

7 On an ordinary desktop configuration, the map-matching and path inference module processes about 50 traces per  
8 second. The runtime required for the travel time estimation increases with the number of trace pairs, days, time-of-  
9 day intervals and network links. For the Stockholm case study, there are 501,555 trace pairs in the data set and the  
10 computations take around 10 minutes. For the whole Singapore network, before merging there are 71,373 nodes and  
11 80,346 links used for map-matching and travel time estimation. Around 900 - 1,250 traces are collected from each  
12 taxi on each day, totalling over 1.01 billion GPS traces. For this data set, map-matching takes around 3 - 4 weeks for  
13 all 59 days of GPS data, and travel time estimation takes around 1-2 weeks.

14 In choice set generation, for fixed network parameters (number of time periods, number of support points,  
15 number of links, number of stochastic links), the runtime of choice set generation is linear in link elimination interval,  
16 number of simulations, and number of OD pairs. After extracting the network, choice set generation takes around 4.7  
17 mins per OD pair in the Stockholm case study, and 1.1 mins per OD pair in the Singapore case study.

### 18 4.1.4 Software

19 Various computer programs and software have been utilized to perform this research. For the Stockholm Case Study,  
20 the map-matching and path inferring programs are coded in Java, and the travel time estimation program is coded  
21 in Matlab. The data is visualized in Google Earth and is stored in PostgreSQL database with PostGIS plugin. For  
22 the Singapore Case Study, computer programs are coded in Python for the map-matching, path inferring, and travel  
23 time estimation. For both case studies, computer programs are coded in C++ with Microsoft Visual Studio for the  
24 ORP algorithm, and choice set generation and evaluation. In the simulation method, the link travel time samples are  
25 generated by R utilizing the normal distribution function. Excel Macros are also utilized throughout the experiment to  
26 prepare the inputs and analyze the results.

## 27 4.2 Stockholm Case Study

### 28 4.2.1 Data

29 As the capital and the largest city of Sweden, Stockholm constitutes the most populated urban area in Scandinavia. As  
30 for transportation network, Stockholm is at the junction of the European routes E4, E18 and E20, and a half-completed  
31 motorway ring road exists on the south and west sides of the City Center. A subset of the Stockholm network is studied,  
32 which includes the Arlanda airport area, E4 motorway between the airport and the city, and northeast part of the inner  
33 city. In this sub-network, according to the observations of local residents, taxi drivers adapt to traffic conditions when  
34 making route choices going into and out of the city center. In particular, between point A and point B shown in Figure  
35 1, there is a choice among two common routes, either the western route along E4 or the eastern route along E18  
36 and LV276. The GPS traces utilized are generated by the taxis from a fleet management system in Stockholm. The  
37 trace generation process is time-based with data from November 1, 2012 through January 18, 2013, covering the time  
38 intervals of Mondays through Fridays, resulting in 56 days (support points).



FIGURE 1 : Road Network in the Stockholm Case Study

#### 1 4.2.2 Choice Set Generation

2 The choice set generation and network data processing methodologies are introduced in Section 2.3 and in Section 3.  
 3 Based on the vehicle trajectories data, the number of GPS traces per trip range from 2 to 47, the departure times span  
 4 from around 6:30 to 9:00 AM.

5 For choice set generation, 997 out of 11,858 OD trips are sampled. The sampling is based on three criteria.  
 6 Firstly, wide spread locations of OD pairs are selected to make the choice sets diverse. Secondly, a long study period  
 7 results in longer runtime; therefore, it is beneficial to select trips in a certain sub-time period such as the peak hour,  
 8 which have concentrated departure times and also enough diversity. Thirdly, longer trips are preferred since they are  
 9 more likely to show strategic route choice behavior of drivers. The actual paths usually contain much more links than  
 10 the scattered GPS traces, since the gaps of the observed traces are generally large. Therefore, the estimated actual  
 11 number of links in the paths is used to select longer trips. Link elimination interval, as introduced in Section 2.3.1, is  
 12 also decided by the estimated actual number of links in the paths for the same reason. In the Stockholm case study,  
 13 the algorithm eliminates every 10 links.

#### 14 4.2.3 Coverage

15 Initially, only the link elimination method is adopted for choice set generation. After evaluation, the coverage is found  
 16 to be relatively low. Possible reasons of the low coverage are described in Section 2.3.1. Thus improvements are  
 17 needed to increase the coverage.

18 Simulation method is then conducted on the unmatched OD pairs. Travel time samples are generated by  
 19 independent normal distribution, with mean as the estimated travel time and standard deviation as 1/4 of the mean.  
 20 In each of the 30 simulations, ORP is calculated for each OD pair and added to the corresponding choice set, if not  
 21 already in it. After evaluation, the coverage is increased greatly to 81% for 100% overlap, and 92% for 100% overlap,  
 22 which shows the effectiveness of simulation. The goal for satisfactory coverage is achieved. Table 2 demonstrates the  
 23 improvement of coverage after adding the simulation method.

#### 24 4.2.4 Routing Policy Choice Sets v.s. Path Choice Sets

25 The routing policy choice set is compared to a benchmark of path choice set based on static and deterministic link travel  
 26 times, as conventionally used for route choice in the literature. In benchmark analysis, the travel times are changed

1 to deterministic and static by taking average of the original link travel times over time periods and support points.  
 2 The static shortest path choice sets are then generated utilizing link elimination and simulation. Table 2 illustrates the  
 3 comparison of coverage between routing policy and static shortest path choice sets. This comparison is an indication  
 4 that the routing policy choice sets could provide better coverage than the path choice sets. However, since we have  
 5 not exhausted all the possible means to improve either type of choice sets, future work is needed to provide a more  
 6 conclusive comparison.

Choice Set Type	Choice Set Generation Method	OD Pairs	Overlap Threshold	Matching OD Pairs	Coverage
Routing Policy Choice Sets	Link Elimination	997	1	633	0.64
		997	0.8	788	0.79
	Link Elimination and Simulation	997	1	803	0.81
		997	0.8	917	0.92
Path Choice Sets	Link Elimination	997	1	563	0.56
		997	0.8	718	0.72
	Link Elimination and Simulation	997	1	583	0.58
		997	0.8	737	0.74

**TABLE 2 :** Comparison of Coverage on Varying Choice Set Types and Methods for the Stockholm Case Study

#### 7 4.2.5 Other Potential Improvements on Coverage

8 Although the coverage is satisfactory, the current algorithm uses only the expected travel time in the calculation of  
 9 ORP. When choosing a route, travelers consider a lot of other important factors such as travel time standard deviation,  
 10 number of left turns, number of intersections, scenery, highway bias, tolls and congestion pricing. To capture varying  
 11 attitudes of travelers toward the highway and intersections, a highway bias and a link number bias are introduced in the  
 12 route choice. The highway bias is implemented by multiplying a certain constant to the highway link travel times. The  
 13 constant is set to be positive and smaller than 1, so that generated policies include more highways as if travelers prefer  
 14 highways, which is the case in real-life. The link number bias is implemented by adding a constant to the travel times  
 15 of intersection links. The constant is set to be positive, so that routing policies with a larger number of intersection  
 16 links are penalized, since travelers generally prefer routes with fewer intersections. In the algorithm, a link is first  
 17 checked if it is an intersection link or a highway link, and then its travel time is added a constant penalty if it is an  
 18 intersection link, and is reduced to a certain proportion if it is a highway link. Both highway bias constant and link  
 19 number constant could be adjusted empirically based on network parameters to reflect the attitude levels of travelers.  
 20 Preliminary tests with the two factors are done resulting in a small increase in coverage, and more testing is necessary  
 21 to draw a conclusion.

#### 22 4.2.6 Adaptiveness

23 Based on the concept of adaptiveness introduced in Section 2.4.2, there are two extreme cases: if a routing policy  
 24 realizes as the same path on all different days, adaptiveness should be 1 divided by 56, which equals 0.018; if a routing  
 25 policy realizes as a different path on all different days, the adaptiveness should be 56 divided by 56, which equals 1.  
 26 The results show that 762 out of 997 OD pairs have adaptiveness bigger than 0.018, indicating that routing policies  
 27 are realized as different paths over days. The average adaptiveness of all routing policy choice sets is 0.098, and the  
 28 median is 0.045. A histogram of the adaptiveness of all OD pairs is shown in Figure 2.

29 The adaptiveness increases with OD expected travel time, as shown in Figure 3. OD expected travel time is  
 30 averaged over all routing policies in the choice set for a given OD pair. This trend is intuitive, as longer trips generally  
 31 motivate travelers to consider more carefully of their travel plans as well as allow for more diversion opportunities.

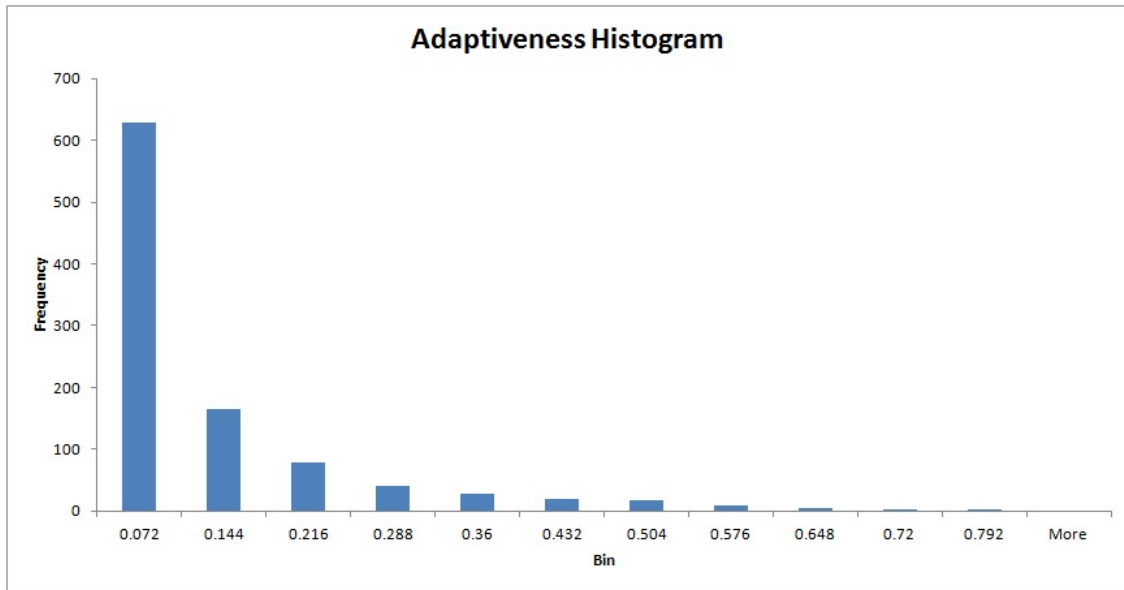


FIGURE 2 : Adaptiveness Histogram for the Stockholm Case Study

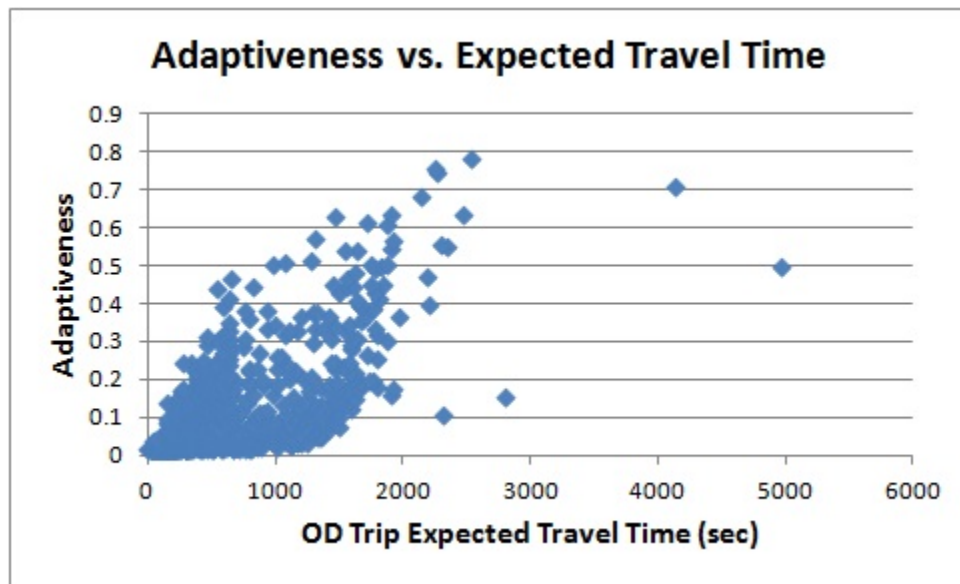


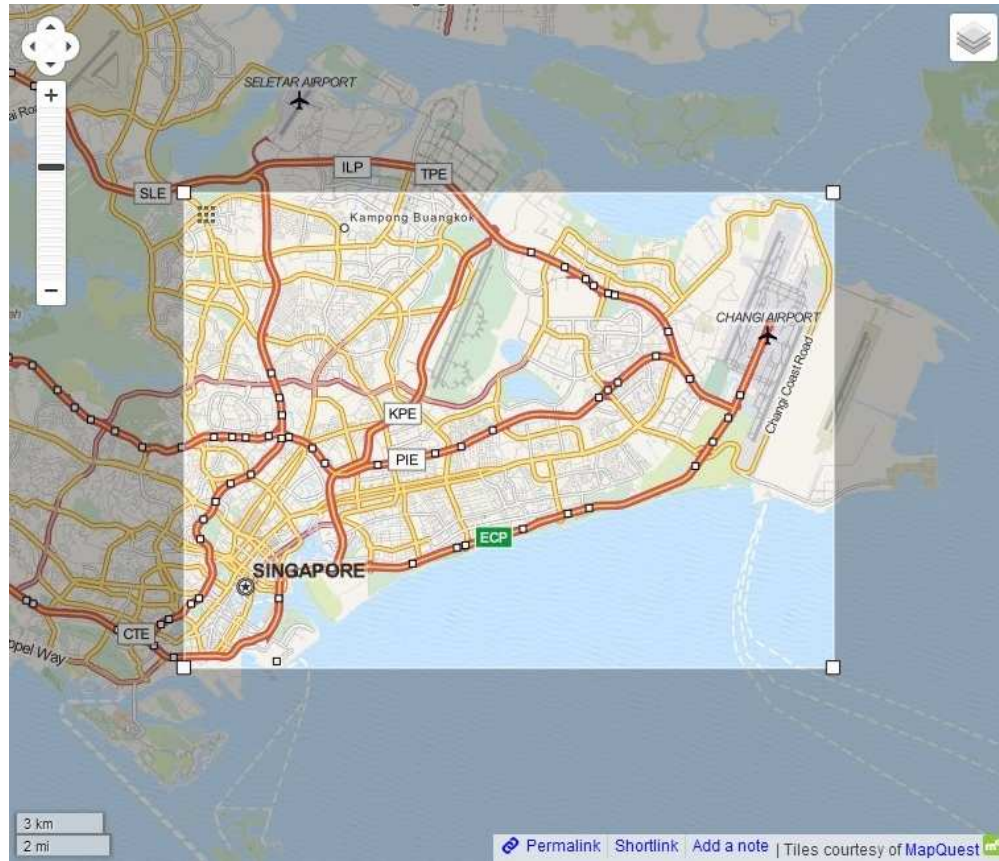
FIGURE 3 : Adaptiveness VS Expected Travel Time for the Stockholm Case Study

### 1 4.3 Singapore Case Study

#### 2 4.3.1 Data

3 A case study is also conducted on a sub-network in Singapore to further illustrate the general applicability of the  
 4 methodology. Again, Taxi GPS data is used. The network is developed from OpenStreetMap (OSM), which is a  
 5 collaboratively created free editable online map of the world. A graphical representation is compiled utilizing the  
 6 OSM network to create nodes, links, and free flow travel times. A sub-network of Singapore is then extracted for  
 7 the implementation of choice set generation and evaluation. This sub-network contains two main areas of interest,

1 including the Changi Airport area and the selected downtown area, as well as the corridor between them, including  
 2 two major freeways and a number of arterial paths (shown in Figure 4).



**FIGURE 4** : The Sub-Network for the Singapore Case Study

3 In order to obtain the vehicle trajectories and link travel times from taxi GPS data, map-matching is conducted  
 4 as described in Section 3. In this method, a total of over 19.35 million hired taxi trips over two months are identified  
 5 whose paths are inferred. Over 2 billion link travel time estimates are then generated. Aggregation is performed for  
 6 the combinations of time intervals, support points, and links. Similar data processing as described in Section 3 is  
 7 conducted to fill in missing values for link-day-interval combinations where there are not enough observations. Such  
 8 links are identified as deterministic.

9 The GPS traces cover the entire day for each taxi, but the frequency of the GPS traces vary greatly. A  
 10 histogram with regards to the time gap between consecutive GPS traces shows that the peak is at 3 minutes (10%), and  
 11 over 95% of the time gaps are shorter than 4 minutes. In the map-matching process, the shortest path connecting two  
 12 candidate links for consecutive GPS traces is used as the edge of the candidate graph. However, for GPS traces with  
 13 a low sampling rate, the shortest path assumption is not very effective. In the process of identifying hired taxi trips, a  
 14 threshold of 4 minutes is set with regards to the time gap between two consecutive GPS traces, i.e., if the trip contains  
 15 two consecutive GPS traces that are more than 4 minutes apart, the trip is dropped. In this way, sufficient trips are kept  
 16 and trips with very sparse GPS traces are removed.

### 17 4.3.2 Choice Set Generation and Evaluation

18 Following the same procedures as in the Stockholm case study, routing policy choice sets are generated for OD pairs  
 19 of the paths inferred from taxi GPS data, first by link elimination and then by simulation, based on estimated link travel  
 20 times. The initial coverage results are relatively low and preliminary investigations have been conducted to explore  
 21 possible reasons.

1 The procedure of the investigations is described as follows. Dense taxi trips are first identified, i.e., trips with  
2 maximum time gap between two consecutive GPS traces shorter than 30 seconds. For those dense trips, it is assumed  
3 that map-matching is effective thus the inferred paths are considered as the ground truth paths. Sampling of the GPS  
4 traces from those dense trips is then conducted, with the sampling rate at one minute, two minutes, three minutes and  
5 four minutes, respectively. Based on the sampling, two tests are carried out.

6 The first test checks whether the ground truth paths are contained in the candidate graph constructed during  
7 the map-matching process for the sampled GPS traces. The purpose of this test is to investigate whether it is effective  
8 to use the shortest path to connect two candidate links for consecutive GPS traces as the edge of the candidate graph.  
9 The results present that the ratios of the ground truth paths contained in the candidate graph for the sampled GPS  
10 traces are 98.4%, 95.6%, 91.9%, and 85.5% for sampling rate at one minute, two minutes, three minutes and four  
11 minutes, respectively. These results indicate that it is effective to use the shortest path to connect two candidate links  
12 for consecutive GPS traces as the edge of the candidate graph.

13 The second test compares the ground truth paths with the paths inferred from the sampled GPS traces. The  
14 purpose of this test is to check the effectiveness of the score function used in the map-matching process. The results  
15 show that the ratios of the ground truth paths matched with the inferred paths for the sampled GPS traces are below  
16 20% for all four sampling rates. These results indicate that the score function is not effective to select the ground truth  
17 paths from the candidate graph. Further tests are conducted with spatial/temporal consistency terms added to the score  
18 function or used as filters. The resultant matching ratios increase to around 70% for all four sampling rates.

19 Map-matching with the updated score function and link travel time estimation with the new inferred paths  
20 are ongoing. Results regarding routing policy choice set generation and evaluation will be presented in the future.

## 21 **5 Conclusions and Future Directions**

22 This paper studies routing policy choice set generation based on individual-level route choice data from GPS observa-  
23 tions in real-life large networks, where the travelers can revise the route choices based upon en route information. Case  
24 studies are carried out in two real-life networks, Stockholm, Sweden and Singapore, based on GPS data from hired  
25 taxis. The link elimination and simulation methods are combined to generate choice sets, where the routing policies  
26 represent alternatives that allow re-routing. The choice sets are evaluated in terms of adaptiveness and coverage. This  
27 research makes a contribution by investigating, for the first time, routing policy choice set generation under real-time  
28 information in real-life large STD networks based on GPS data. For the Stockholm case study, an adequate coverage  
29 of 92% for 80% overlap threshold is achieved through a combination of link elimination and simulation. There is also  
30 an indication that routing policy choice sets provide better coverage than path choice sets. For the Singapore case  
31 study, routing policy choice set generation is on-going and further improvements in coverage are needed.

32 In future studies, the coverage will be improved by using better map-matching and adding other attributes  
33 to the ORP algorithm including highway bias and link number bias. Since the advances in GPS technology make it  
34 possible to track individual route choices of travelers, GPS taxi data is utilized for this route choice study. However,  
35 this type of passive monitoring cannot generate observations regarding the information access of travelers, and the  
36 fact that the information in a dynamic network changes over time and space makes it more difficult. This study  
37 circumvents this difficulty by assuming POI access, but more realistic information situations are to be explored in  
38 future studies. Drivers' information access can be obtained through a number of means, for example, periodic survey  
39 questions delivered to smartphones in real time after some major diversion points are traversed, location prompted  
40 recall survey at the end of day, and records of in-vehicle GPS navigation systems that provide real-time information.

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