Automatic Deduction in an AI Geometry Book
(SHORT-ST: special track on Intelligent Documents)

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Abstract. The pursuit of an *AI Geometry Book* should involve the study of how currently developing methodologies and technologies of geometry knowledge representation, management, deduction and discovery can be incorporated effectively into a computational application, a "book" of the future.

In the geometry book of the future statements and proofs should be enlighted by dynamic geometry sketches and diagrams, and the correctness of the proofs should be ensured by computer checking. The book will be intelligent, the reader should be able to ask closed or open questions, and can also ask for proof hints. The book should also provide interactive exercises with automatic correction.

To fulfil such a goal the development of an open library of geometry automated theorem provers with a carefully design application interface protocol, must be considered. This would allow to link computer platforms for geometry with theorem provers, providing the automatic deduction capabilities for the *AI Geometry Book*.

1 Introduction

The geometry book of the future should be intelligent, correct, visual, adaptive, and collaborative in its production and use. Although the previous efforts and achievements help set aside many obstacles on the way to an intelligent geometry book, all the approaches, methods, techniques, basic tools, and systems, are still developed, applied, and conceived in separated, and relatively small, circles [13,19].

When considering the application of automated reasoning in a learning context two, somehow opposing, goals are to be considered: efficiency and readability of the proofs. Whenever the help of the computer is considered, the user wants a fast and friendly answer [6].

Overall, the existing methods for automated theorem proving and discovering in geometry are very efficient and, in some cases, providing an output that can be used in a learning context. However, finding a method/implementation capable of both (fast and friendly) is only possible for a small number of cases. There is still a lot of room for further improvements [3]

Automated deduction methods can be more tightly integrated within interactive theorem provers and dynamic geometry [8,9]. Automated deductions methods are currently unable to produce guidance and explanations for educational

purposes. New techniques are needed involving synthetic geometry, formalisation techniques and artificial intelligence [2,4,5,16].

Computer checking of geometric proofs has been addressed in the literature but: current formal proofs are not readable, natural and visual languages should be connected with formal deductions; several foundations of geometry and automated theorem provers are available, but all connections between them have not been obtained; the editing of a pedagogical corpus from formalisation have not been studied deeply enough [4,7,10,20,21].

Building on the work already developed in the systems: *OpenGeoProver* (*OpenGeoProver*) an open source project implementation of GATPs [1,2]; the *Thousand of Geometric problems for geometric Theorem Provers (TGTP)* a problem repository and test bench [12]; and the standardised format for constructions and conjectures in geometry, I2GATP [14], we intend to extend and integrate these projects, providing deductive services to other applications.

The immediate program include the development/implementation of new methods to incorporate into the *OpenGeoProver*; finalising the standard format I2GATP to be able to define a programming protocol between *OpenGeoProver* and third-party programs; extend the platform *TGTP* in such a way that it can become the base for a competition between GATPs developed by the scientific community, to boost the improvement of methods/implementations.

Overview of the Paper This paper is organised as follows: In section 2 the current status and future developments of *OpenGeoProver* are described. In Section 3, the *TGTP* platform is described and the goal for a future competition between GATPs are set. In Section 4 the common format I2GATP is described and future developments are foreseen. In Section 5 the integration issues are discussed. In Section 6 some final remarks are drawn.

2   Open Geometry Prover

The *OpenGeoProver*[1] is an open source project, aiming to implement various geometry automated theorem provers (GATPs). It can be used as a stand-alone tool but can also be integrated into other geometry tools, such as dynamic geometry software, e.g., work is being made to integrate *OpenGeoProver* with *GeoGebra* [2]. In its current state, *OpenGeoProver* implements two algebraic methods, the Wu's method and the Gröbner basis method, as well as one semi-synthetic method, the *area method*. Some initial work on the implementation of the *full-angle method* was already done [1].

As future developments we have: the implementation of new methods, finishing the work done with *full-angle method*, implementation of the deductive databases method, among others; the design of an application programming interface (API), allowing the different implemented GATPs to be easily used by other programs; write an extensive documentation, with a special care in the description of the API.

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[1] https://github.com/ivan-z-petrovic/open-geo-prover/

The *OpenGeoProver* will constitute an excellent mean to provide automated reasoning resources to the *AI Geometry Book*.

3   The *TGTP* Platform

Thousands of Geometric problems for geometric Theorem Provers (*TGTP*) is a Web-based library of problems in geometry, a comprehensive common library of problems with a significant size and unambiguous reference mechanism, easily accessible to all researchers in the automated reasoning in geometry community. It share the database of problems with the *GeoThms*, a Web-based framework for exploring geometrical knowledge that integrates Dynamic Geometry Software (DGS), Automatic Theorem Provers (ATP), and a repository of geometrical constructions, figures and proofs [5,12].

One of the main motivations in building *TGTP* is to support the testing and evaluation of geometric automated theorem proving (GATP) systems. Providing a common library of problems for the testing of GATPs it will allow the pursue of the development of better methods/implementations.

Along the lines of the competition CASC [18], the conception and operationalization of a competition among GATPs is part of the future developments for the *TGTP*, it will allow the development of better methods/implementations, but will also push in the direction of a better interface between the geometric information and the programs that can use the information.

For the *AI Geometry Book* a repository such as *TGTP* is of extreme importance, it will constitute a source of valuable information to enrich the "book".

4   The I2GATP Common Format

The I2GATP format is an extension of the I2G (*Intergeo*) common format aimed to support conjectures and proofs produced by geometric automatic theorem provers. The goal in building such a format is to provide a communication channel between different tools from the field of geometry, allowing linking such tools, as well as allowing the use of geometric knowledge kept in different repositories [14,17].

The I2GATP format is a combination of four XSD files: `information.xsd`, with all the meta-information regarding the conjecture; `intergeo.xsd`, the *Intergeo* I2G format, for the description of geometric constructions; `conjecture.xsd`, the translation of `geocons.dtd` to the XSD format [11]; `proofInfo.xsd`, the meta-information regarding the proof generated by a given GATP on a given computing platform..

The I2GATP container is an extension of the I2G container. In addition to the information in the I2G container, all the information regarding the geometric conjecture and all the proofs attempts are kept in the I2GATP container.

The I2GATP library is an open source project[2], to support the writing of filters from/to the I2GATP container and different geometric computation tools.

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[2] `https://github.com/GeoTiles/libI2GATP`

The I2GATP library will enable to specify an API, opening the possibility of geometric information interchange and the use of the GATPs in *OpenGeoProver* (and others, supporting the I2GATP format) and also the information contained in *TGTP*.

5   Deductive Tools in an AI Geometry Book

The integration that can be seen in systems like *GeoThms*, *GeoGebra*, *Cinderella*, *JGEX*, *GCLC* and *GeoProof* [2,4,5,10,16,21], should be used as an inspiration to the *AI Geometry Book*.

The *AI Geometry Book* should be able to interface with repositories of geometric information and with DGSs and GATPs. Using the I2GATP common format as interface we could include deductive services via the *OpenGeoProver* and information via the *TGTP* repository.

Another twodevelopment that the author and other researcher are working on are: a taxonomy for geometric problems allowing to adjust the queries to each and every user [15]; a semantic search mechanism that will allow to search for geometric information in a geometric fashion.[3]

6   Conclusions & Future Work

The pursuit of an *AI Geometry Book*—where the "book" is understood as a computation platform that will extend the concept of (non-digital) object of study, a repository of (non-digital) knowledge, to the digital world—needs the use of many AI techniques. AI is needed for the automatic deduction, for the adaptation to the user's profiles, for the search of information.

Some of this issue are already, partially, solved, e.g. there are already many efficient GATPs, but the integration of all the "players" in such a way that a *AI Geometry Book* can be built is still to be done. As described above we are working on those issues, trying to integrate *OpenGeoProver* with *TGTP* through the I2GATP common format, and also on some issued related with the usability of such integrated platform, i.e. the adaptability and the semantic geometric search.

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[3] Geometric Figure Mining via Conceptual Graphs, Yannis Haralambous and Pedro Quaresma, submitted to ADG2018.

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