What is the Cost of the Index Selector Task for OFDM with Index Modulation?

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Abstract—Index Modulation (IM) is a technique that activate k out of n subcarriers of an OFDM symbol to transmit $p_1 = \lfloor \log_2 \binom{n}{k} \rfloor$ bits in symbol's indexes. Since both the symbol's spectrum width and transmission air-time duration remain the same, OFDM-IM outperforms OFDM's Spectral Efficiency (SE) for larger values of $\binom{n}{k}$. However, OFDM-IM requires an *extra* step called Index Selector (IxS) which takes T_α time units to map a given p_1-bit input to its corresponding pattern of active subcarriers. This *extra* overhead virtually enlarges the symbol duration, which is not captured by the classic SE definition. To fulfill this gap, in this work we present the Spectro-Computational Efficiency (SCE) metric. SCE parameterizes either the absolute runtime of T_α on a reference hardware or its computational complexity $T_\alpha(n, k)$ as function of n and k. Based on SCE, we present theoretical case studies to identify the asymptotic bounds for $T_\alpha(n, k)$ across different choices of k. if $T_\alpha(n, n/2)$ is at most linear on n the resulting overhead is asymptotically negligible and IxS can handle an arbitrarily large OFDM symbol. Otherwise, OFDM-IM's SCE tends to zero regardless of the hardware processor speed. Also, we situate the inflection-point values for OFDM-IM's SCE between $\binom{6}{3}$ and $\binom{14}{7}$ in some practical case studies.

Index Terms—OFDM, Index Modulation, Spectral Efficiency, Computational Complexity, Spectro-Computational Efficiency.

I. INTRODUCTION

Index modulation (IM) is a PHY layer technique that relies on combinatorics to leverage the overall distinct waveform realizations of an OFDM symbol. In classical N-subcarrier OFDM, each particular subcarrier is independently modulated by an M-point constellation diagram. The *IM principle* adds a novel configuration parameter $k < N$ to OFDM, which stands for the number of subcarriers that must be 'active' in a symbol. Combinatorial theory tells us that there exist $\binom{N}{k} = N!/(k!(N-k)!)$ ways to activate k out of N subcarriers. Each of these resulting subcarrier activation pattern (SAP)

can be used to transmit $p_1 = \lfloor \log_2 \binom{N}{k} \rfloor$ bits. This way, the successful demodulation of those p_1 bits does not depend on the waveform of the active subcarriers but only on the correct detection of the ON/OFF status of all N subcarriers.

First attempts to apply the IM principle for OFDM date back to the 90s [1]. However, the computational complexity to map and detect the possible SAPs of a typical standard OFDM symbol may be unfeasible in practice. A landmark answer to this limitation was accomplished with OFDM-IM [2]. Instead of applying IM to the entire N-subcarrier symbol, OFDM-IM subdivides it into G smaller subblocks, each consisted of $n = \lfloor N/G \rfloor$ subcarriers. This way each subblock is independently modulated with $p_1 = \lfloor \log_2 \binom{n}{k} \rfloor$ bits, and the detection search space decreases. Despite that reduction, an *optimal* detector still has to iterate across a combinatorial search space on n and k. To overcome that, the authors present a detector that selects the k strongest received subcarriers as active and sets the remaining $n - k$ as inactive. Because the strategy avoids checking all SAPs to save time it is sub-optimal. However, the authors reported several n and k values under which OFDM-IM outperforms the bit-error rate (BER) of OFDM under the same asymptotic computational complexity of detection.

A. Related Work

The subcarrier deactivation strategy of OFDM-IM has shown to overcome OFDM when BPSK modulation is set (i.e., $M = 2$) [3]. As M becomes larger, OFDM-IM sacrifices $\log_2 M$ bits per deactivated subcarrier, so it is outperformed by OFDM. To handle this problem, novel strategies have been proposed on top of OFDM-IM to increase its waveform realizations. Fan et al. [4], [5], for instance, generalizes OFDM-IM to support variable k per subblock. Mao et al. [6] present the Dual-Mode (DM) OFDM-IM in which all subcarriers are active. Thus, all n subcarriers send data individually. The distinction between "ON" and "OFF" subcarriers happens in a logical way by modulating them with specific constellation sets. DM-OFDM-IM was generalized to support variable k in [7]. Wen et al. [8] present the 'Multiple Mode' OFDM-IM design in which the group of distinguishable constellations is made arbitrary. This resembles a kind of DM-OFDM-IM in which subcarriers can be classified into groups other than just

"ON" or "OFF". MM-OFDM-IM is generalized for variable k in [9]. All these proposals focus on achieving a good balance between BER and detection computational complexity because of the enlarged SAP space they achieve.

Although detection represents the major change IM demands on classic OFDM [10], [11], it is not the unique. The OFDM-IM family adds an extra step to the OFDM transmission chain[1] called *Index Selector* (Fig. 1). IxS changes the original OFDM signal constellation mapper, which is responsible to generate the N complex constellation points that modulate their N respective subcarriers. In particular, IxS takes p_1 bits as input and returns the k subcarriers indexes that must be active in a subblock. Each active subcarrier can be individually modulated by an M-size constellation yielding more $p_2 = k \log_2 M$ bits. This is illustrated in Fig. 1 assuming one subblock, i.e., $G = 1$ and $n = N$. For small waveform realizations, the IxS task can be efficiently performed in $O(1)$ time by means of an $\binom{n}{k}$-entry Look-Up Table (LUT). However, LUTs growth becomes prohibitive for larger values of n and k.

To avoid storing all $\binom{n}{k}$ SAPs, OFDM-IM proposals [2]–[9], [13], [14] rely on an online IxS to compute SAPs on-the-fly. However, this is achieved at the penalty of an overhead not present in classic OFDM, which may compromise the benefit of OFDM-IM for larger values of n and k. Nonetheless, as far as we know, no prior work concerns on studying this issue.

B. Our Contribution

In this work we present the Spectro-Computational Efficiency (SCE) metric, a novel PHY layer performance indicator. Following classical SE formula [12], OFDM-IM proposals usually calculate SE as the ratio between bit rate to the consumed spectrum. Since OFDM-IM symbols consume the same amount of spectrum and (the *over-the-air*) time of OFDM, prior OFDM-IM SE formulas focus only on reflecting the total number of bits they manage to transmit in the channel[2]. Besides these parameters, SCE also parameterizes the computational complexity required to build the symbol. *SCE suits to evaluate PHY layer enhancements that demand extra computational resources to improve bit rate of prior designs*. As illustrated in Fig. 1, this is the case of OFDM-IM (with respect to OFDM) because of IxS. Our metric enables one to answer whether the increase in the computational complexity pays off for the extra bits achieved.

Based on SCE we present theoretical and practical case studies. In our theoretical case study we identify the IxS asymptotic bound across different formulas for k. In particular for $k = n/2$ (the value that maximizes the number of transmitted bits p_1 [3]), our theoretical study shows that the IxS's computational complexity must be $O(n)$ otherwise the OFDM-IM's SCE tends to zero for larger n and k. In other words, the resulting runtime overhead does not pay off for the OFDM-IM bit gain. By contrast, *if the IxS's complexity is*

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[1]please, for further details about the steps of an OFDM transmitter, refer to DSP textbooks e.g. [12].

[2]Mao et al. [10] survey the SE formulas of several OFDM-IM proposals.

$O(n)$, *the resulting overhead is asymptotically negligible and the OFDM-IM mapping can handle a single arbitrarily large subblock*. In our practical study we situate the best OFDM-IM CSE between $\binom{6}{3}$ and $\binom{14}{7}$.

The remainder of this work is organized as follows. In Section II we present the SCE metric. In Section III we study OFDM-IM's SCE to identify the ideal asymptotic bounds for the IxS's computational complexity. We also present a theoretical case study based on a particular IxS algorithm adopted by the IM literature. In Section IV we present a practical case study to assess OFDM-IM's SCE on real hardware. Finally, in Section V we present conclusion and future directions.

II. THE SPECTRO-COMPUTATIONAL EFFICIENCY METRIC

The Spectral Efficiency (SE) is a popular metric to measure the performance of a PHY layer design. In digital modulation systems, SE (bits/seconds/Hertz) is usually defined as the ratio between the bit rate R to the bandwidth W [12]. In OFDM, $R = B_{OFDM}/T_N$, in which B_{OFDM} and T_N are the number of bits in the N-subcarrier symbol and its duration time, respectively. To combat multipath fading, the lasts $T_{cp} < T_N$ time samples of the symbol are copied to its beginning. This is the so-called Cyclic Prefix (CP), a.k.a. inter-symbol Guard Interval (GI). Translated to the frequency domain, T_{cp} corresponds to N_{cp} subcarriers, and the following relationship holds $T_{cp} = (N_{cp}/N)T_N$. Therefore, the overall OFDM symbol duration enlarges to $T_{OFDM} = T_N + T_{cp}$ yielding the effective SE shown in Eq. 1.

$$SE_{OFDM} = \frac{(B_{OFDM}/T_{OFDM})}{W} \qquad (1)$$

$$= \frac{B_{OFDM}}{N + N_{cp}} \qquad (2)$$

$$B_{OFDM} = N \log_2(M) \qquad (3)$$

Considering the equality $W = \Delta f N$ (Hz), where Δf stands for inter-subcarrier space (Hz), the orthogonality condition of OFDM, i.e. $T_N = 1/\Delta f$ (secs) and M-point modulation (Eq. 3), Eq. 1 rewrites as Eq. 2.

Basar et al. [2] was the first to employ Eq. 2 to compare OFDM and OFDM-IM performance. Ever since, the formula has served as base for comparative studies between OFDM and multicarrier IM proposals [2]. As one can observe in Table III of the survey work [10], the significant change in comparison to the OFDM SE formula is to reflect the bit gain each respective IM proposal achieves with the IM technique.

A. Spectral Efficiency Revisited by Computational Overhead

Equation 2 measures SE assuming that both OFDM and OFDM-IM symbols occupy the bandwidth W by the same amount of time $T_N + T_{cp}$. This is accurate if one assumes that both proposals take the same time building their respective symbols. By contrast, any *extra computational step* added to the classic OFDM block diagram to transmit a larger quantity $B_{OFDM-IM}$ of bits might impose a time delay T_α (sec) to the start of the symbol transmission over-the-air. If T_α grows proportionally to $B_{OFDM-IM}$, the resulting overhead could
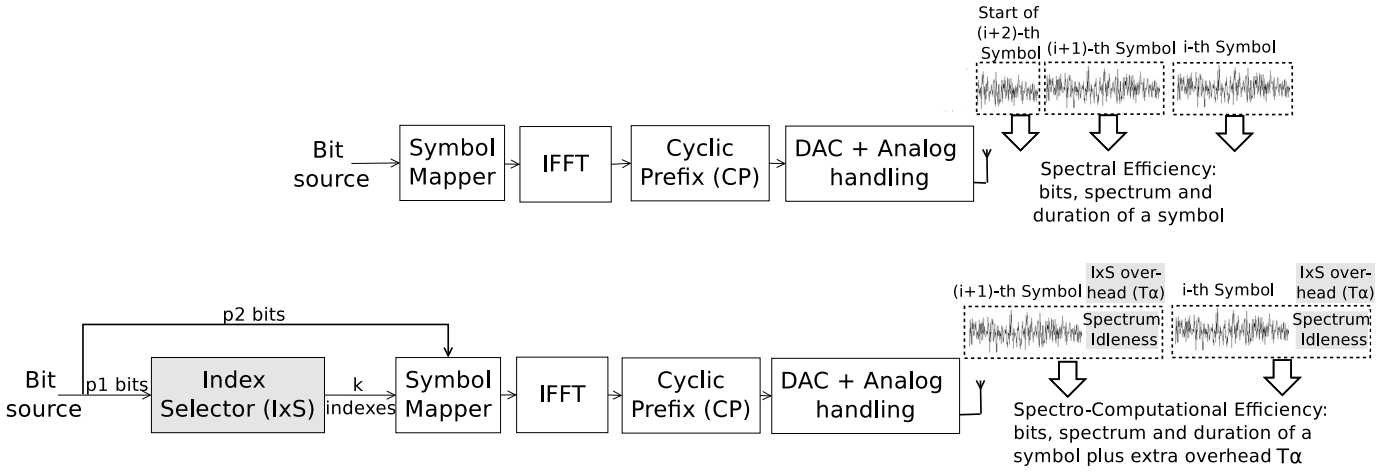
Fig. 1. OFDM-IM (bottom diagram) adds index selector (IxS) to the OFDM signal transmission path (upper diagram). IxS represents $p_1 = \lfloor \log_2 \binom{n}{k} \rfloor$ bits with the activation of only k (out of $n > k$) subcarrier indexes. Assuming an M-point modulation, the active subcarriers transmit more $p_2 = k \log_2 M$ bits. The resulting IxS runtime overhead T_α grows on n and k but is not reflected in prior spectral efficiency (SE) metrics [10]. We propose the Spectro-Computational efficiency metric (SCE) to discover theoretical asymptotic bounds under which T_α is negligible for a single arbitrarily large OFDM-IM subblock.

neutralize the expected SE improvement. Thus, one should study the asymptotic relation between $B_{OFDM\text{-}IM}$ and T_α to identify the region of effective SE gain of the novel proposal.

The classic SE formula does not concern on the impact of T_α on SE. To fulfill this gap we present the Spectro-Computational Efficiency (SCE) metric. Eq. 4 presents SCE for an arbitrary PHY layer whose symbol transmits B bits during T_{SYMBOL} time units. Note that it is the same as for SE unless by the parameter T_α.

$$SCE = \frac{B/(T_{SYMBOL} + T_\alpha)}{W} \qquad (4)$$

The corresponding SCE formula for a PHY layer that enhances OFDM is readily obtained by adding T_α to Eq. 1. As with the cyclic prefix, one can express T_α in the frequency domain to write the CSE OFDM formula (Eq. 6) that corresponds to Eq. 2. If T_α results from an extra procedure whose runtime grows on N, then it can be expressed as function N_α of the number of subcarriers. N_α represents the spectrum idleness period imposed by T_α (Fig. 1).

$$SCE_{OFDM\text{-}FLAVOUR} = \frac{B/(T_{OFDM} + T_\alpha)}{W} \qquad (5)$$

$$= \frac{B}{N + N_{cp} + N_\alpha} \qquad (6)$$

The particular SCE for OFDM-IM is given in Eq. 7. The number of bits $B_{OFDM\text{-}IM}$ (Eq. 8) considers an M-point constellation for the modulation of the k active subcarriers and that the symbol is divided into G n-subcarrier subblocks. In Eq. 9, N_α is a function of n and k, which accounts for spectrum resources of the subblocks.

$$SCE_{OFDM\text{-}IM} = \frac{B_{OFDM\text{-}IM}}{N(n,k)} \qquad (7)$$

$$B_{OFDM\text{-}IM} = G\left(k \log_2 M + \left\lfloor \log_2 \binom{n}{k} \right\rfloor\right) \qquad (8)$$

$$N(n,k) = N + N_{cp} + N_\alpha(n,k) \qquad (9)$$

B. How to Compare PHY Layer Designs with the SCE Metric?

For practical PHY layer comparative case studies T_α shall model the exceeding absolute runtime a novel PHY layer design requires to improve bit rate over its reference counterpart. By 'reference PHY layer design' we mean that all computational steps of OFDM are present in its enhanced version. Thus $T_\alpha = 0$ for the reference design. This means that the SCE is equal to the classic spectral efficiency for OFDM.

SCE enables one to identify whether the extra computational overhead T_α is not asymptotically negligible. This may happen if T_α grows proportionally to the claimed bit rate gain. For example, in the practical case study of Section IV, we model T_α as the mean runtime the OFDM-IM IxS step adds to the OFDM reference design (in which IxS is not needed). In this context, the amount of computational resources required by IxS grows asymptotically over the number $p_1 = \lfloor \log_2 \binom{n}{k} \rfloor$ of bits mapped to the symbol indexes. Thus, the resulting OFDM-IM performance can be affected by such computational complexity. In Section III we evaluate the asymptotic SCE of OFDM-IM considering the asymptotic complexity of different implementations of IxS.

III. ASYMPTOTIC SPECTRO-COMPUTATIONAL EFFICIENCY OF OFDM-IM

In OFDM-IM proposals, the subblock size parameter n can represent a trade-off. On one hand, the number p_1 of bits modulated on the symbol indexes grows for larger n. On the other hand, the IxS computational complexity to select the k out of n indexes grows accordingly. This trade-off also impact the signal detection procedure at the OFDM-IM receiver. However, previous works manage to provide OFDM-IM's detection with the same computational complexity of OFDM's [10], [11]. Differently from the detection step, classical OFDM does not require IxS. Thus, IxS's overhead is inherent to OFDM-

IM and is proportional to n and k. In face of that, prior work recommend not to apply IM technique on all N subcarriers of a symbol [2]. The survey work [11] suggests n to be set with sizes of 2, 4, 6, 8, 32 or 64 subcarriers. As far as we know, no prior work dedicates to study the impact of the IxS's overhead on the OFDM-IM performance.

A. Asymptotic Condition for the OFDM-IM Subblock Size

In this section we focus on a generic recommendation regarding the IxS trade-off, namely, *what is the worst asymptotic complexity $N_\alpha(n,k)$ of a particular IxS implementation that pays off for the bit gain of OFDM-IM?* To this end, we study the asymptotic SCE growth of OFDM-IM (Eq. 7) having n as the key variable. To reach the limit of the OFDM-IM SCE performance, we do $n \to \infty$ assuming a single arbitrarily large subblock, i.e., $N = n$ and $G = 1$. This leads to our Lemma 1.

Lemma 1 (OFDM-IM Spectro-Computational Condition)**.** *The index selector's computational complexity $N_\alpha(n,k)$ OFDM-IM adds to OFDM does not pay off for its bit efficiency $B_{OFDM\text{-}IM}$ unless the Spectro-Computational Efficiency tends to a non-zero positive constant for an arbitrarily large number of subcarriers n. Formally, OFDM-IM shall satisfy inequality 10.*

$$\lim_{N=n\to\infty} \frac{B_{OFDM\text{-}IM}}{N + N_{cp} + N_\alpha(n,k)} > 0 \qquad (10)$$

Proof. If one gives up OFDM-IM in favor of OFDM, then $N_\alpha(n,k) = 0$ and the resulting SCE matches SE (Eq. 2). Recalling that M is constant with respect to N and that $N_{cp} = N/c$ for some constant $c > 0$, the OFDM SCE limit is larger than zero (inequality 11). Therefore, OFDM-IM cannot outperform OFDM's SCE unless the inequality 10 does hold.

$$\lim_{N\to\infty} \frac{O(N)}{N + O(N) + 0} > 0 \qquad (11)$$

$\square$

Aside from IxS, the other computational relevant step of OFDM-IM with respect to OFDM is signal detection. If one considers a detector with the same computational complexity as that of the classical OFDM [10], [11], only IxS's complexity matters. In this context, inequality 11 becomes the necessary and sufficient condition under which OFDM-IM outperforms OFDM in terms of spectro(-computational) efficiency.

Lemma 1 is readily satisfied if one implements IxS as LUTs, i.e. $N_\alpha(n,k) = \Theta(1)$. However, if one defines 'extra computation resources' as complexity of space rather than time, $N_\alpha(n,k) = O(\binom{n}{k})$. In this context, the inequality could not be satisfied unless $B_{OFDM\text{-}IM} = \Omega(\binom{n}{k})$. The key condition to check whether a given OFDM-IM implementation meets Lemma 1 is the asymptotic relationship between $N_\alpha(n,k)$ and $B_{OFDM\text{-}IM}$. This leads to our Lemma 2.

Lemma 2 (OFDM-IM Index Selector Asymptotic Bound)**.** *Lemma 1 cannot be satisfied unless $N_\alpha(n,k) = O(B_{OFDM\text{-}IM})$.*

Proof. Recalling that $N_{cp} = O(N)$, $N = n$ ($G = 1$) and $\log_2 M$ is constant, Ineq. 10 rewrites as Ineq. 12. If $N_\alpha(n,k)$ grows faster than $O(B_{OFDM\text{-}IM})$ Ineq. 12 fails.

$$\lim_{n\to\infty} \frac{B_{OFDM\text{-}IM}}{n + O(n) + N_\alpha(n,k)} > 0 \qquad (12)$$

$\square$

B. What Should Be the IxS Computational Complexity?

Lemma 2 tells us that we can 'look' at $B_{OFDM\text{-}IM}$ to identify the computational complexity IxS shall have in the worst case. The order of growth of $B_{OFDM\text{-}IM}$ depends on the asymptotic relationship between k and n. Assuming $k = n/2 = O(n)$, one can determine the value under which $B_{OFDM\text{-}IM}$ is maximum [3]. Theorem 1 uses this assumption to find out the asymptotic complexity under which the OFDM-IM trade-off can be asymptotically solved.

Theorem 1 (OFDM-IM Free of the Index Selector Trade-Off)**.** *If $k = n/2$, then the IxS computational complexity $N_\alpha(n,k)$ shall be $O(n)$, otherwise the OFDM-IM Spectro-Computational Efficiency tends to zero for arbitrarily large n.*

Proof. From Lemma 2 with $k = n/2$ (and recalling Eq. 8), one gets the relation 13. $\binom{n}{n/2}$ is the so-called central binomial coefficient whose well-known asymptotic growth is $O(2^n/\sqrt{n})$ [15]. From this, one gets relation 14 so 15.

$$N_\alpha(n, n/2) = O\left(n/2 + \log_2 \binom{n}{n/2}\right) \qquad (13)$$
$$= O\left(n/2 + \log_2 (2^n/\sqrt{n})\right) \qquad (14)$$
$$= O\left(n/2 + n - 0.5\log_2 n\right)$$
$$N_\alpha(n, n/2) = O(n) \qquad (15)$$

$\square$

Theorem 1 tells us that hardware improvement has minor positive impact on the SCE asymptotic performance of OFDM-IM if the IxS complexity is not $O(n)$. Of course, one may design an IxS hardware tailored for a given value of n (e.g., Application Specific Integrated Circuit, ASIC). However, to benefit from an arbitrarily large amount n of spectrum, the OFDM-IM IxS complexity should be at most linear on n. *These conclusions cannot be derived from the classic SE formula.* In fact, since SE does not capture the computational complexity, it predicts an increasing performance for OFDM-IM for larger and larger n.

Theorem 1 can also serve as guide to identify ideal $N_\alpha(n,k)$ for different choices of k. For instance, if one chooses $k = \sqrt{n}$ rather than the ideal $k = n/2$, we identify that $N_\alpha(n,k)$ should be $O(\sqrt{n}\log_2 n)$. This means that the IxS algorithm complexity must be asymptotically faster (from $O(n)$ to $O(\sqrt{n}\log_2 n)$) to compensate for decreasing k from $O(n)$ to $O(\sqrt{n})$. Similarly, if one keeps k constant regardless of n (i.e., $k = \Theta(1)$), we verify that the IxS complexity must be $O(\log_2 n)$. Table I summarizes these recommendations.

Asymptotic formula for k		Required bound for the IxS Computational Complexity
Formula	Example	
$\Theta(1)$	$k = 4$	$O(\log_2 n)$
$\Theta(\sqrt{n})$	$k = \lfloor \sqrt{n} \rfloor$	$O(\sqrt{n} \log_2 n)$
$\Theta(n)$	$k = \lfloor n/2 \rfloor$	$O(n)$

C. Theoretical Case Study: Combinadic algorithm

In this section we study the theoretical SCE of OFDM-IM considering the "Combinadic" algorithm [16], [17]. In particular, the implementation of [17] is usually (cross)cited by the OFDM-IM literature as a way of implementing an online IxS [2]–[7], [13], [14][3]. The Combinadic algorithm relies on the fact that each decimal number X in the integer range $[0, \binom{n}{k} - 1]$ has a unique representation $(c_k, \cdots, c_2, c_1)$ in the combinatorial number system [18] (Eq. 16). For OFDM-IM, X represents the p_1 input bits (in base-10) and the coefficients $c_k > \cdots > c_2 > c_1 \geq 0$ represent the indexes of the k subcarriers that must be active in the subblock.

$$X = \binom{c_k}{k} + \cdots + \binom{c_2}{2} + \binom{c_1}{1} \qquad (16)$$

1) Combinadic functioning: The Combinadic algorithm is shown in Fig. 2. Considering that n, k and X are input parameters, the algorithm aims to find out the value for each coefficient c_i, $i \in [1, k]$ such that the Eq. 16 holds. The *candidate* values for the coefficients are $0, 1, \cdots, n-1$, which represent the indexes of the n subcarriers. Of these values, only k will be selected to determine which subcarriers must be active. Starting from $i = k$ until $i = 1$, the i-th iteration of the outer loop in Fig. 2 determines the value for the i-th coefficient c_i, $i \in [1, k]$. Once the outer loop selects the coefficient to be computed, the inner loop employs a greedy approach to determine its value. In its first round, Combinadic determines the value for c_k. To this end, it assigns c_k with the largest candidate value $n - 1$ and checks whether $\binom{c_k}{k} < X$ holds. If this logic test fails, the inner loop keeps decrementing c_k until the test passes. When this happens, X is decremented by $\binom{c_k}{k}$. Also, the largest candidate value for the next coefficient c_{k-1} is determined considering that no two coefficients has the same value and $c_k > c_{k-1}$. Then, the process repeats until all remainder coefficients are determined.

2) Combinadic time complexity: In a particular *worst-case* instance, the inner loop test fails for $n - 1, n - 2, \cdots, k$ in the fist round. Thus c_k is assigned to $k - 1$. This narrows the list of candidates (for the remainder $k - 1$ coefficients) to the values $k - 2, k - 3, \cdots, 1, 0$. In fact, since the combinatorial number system ensures that all k coefficients are distinct and that c_k is the largest one, a candidate value that fails for c_k can be discarded for c_{k-1} and so on. Thus, after c_k is determined, there must be at least $k - 1$ candidate values for the remainder

[3]the MM-OFDM-IM family employs a similar algorithm known as factoradic [8], [9].

```
{X, k and n are input parameters. Array c is returned};
largestCandidate ← n − 1;
for i = k downto 1 do
    c_i ← largestCandidate;
    while (c_i choose i) > X do
        c_i ← c_i − 1;
    end while
    X ← X − (c_i choose i);
    largestCandidate ← c_i − 1;
end for
```

Fig. 2. Asymptotic dominant code snippet of the Combinadic algorithm [16], [17]. The input X is mapped into k coefficients c_i ($1 \leq i \leq k$), each representing a subcarrier index to be active in the symbol. Worst-case complexity is $O(nk)$, i.e. $O(n^2)$ for $k = n/2$.

$k - 1$ coefficients. Because of this, there is only one logic test per candidate value in the inner loop regardless of the number of coefficients. Since there are n candidate values, the inner loop takes $O(n)$ time regardless of the outer loop. In each test of the inner loop, Combinadic relies on the multiplicative identity (Eq. 17) to compute the binomial coefficient value in $O(k)$ time.

$$\binom{n}{k} = \prod_{i=1}^{k} \frac{n - i + 1}{i} \qquad (17)$$

Therefore, the overall computational complexity of the Combinadic algorithm [17] is $O(nk)$. Considering $\binom{n}{n/2}$, which represents the largest binomial coefficient hence, the maximum bit gain of OFDM-IM [3], the complexity becomes $O(n^2)$.

Because the Combinadic's runtime complexity does not meet our Theorem 1, *the SCE gain resulted from hardware improvement has negligible effect on OFDM-IM for sufficiently large n*. This is illustrated on Fig. 3 considering some illustrative hardware speedups that attenuate Combinadic runtime by $200\times$ and $300\times$, i.e., $n^2/200$, $n^2/300$, respectively. By contrast, a hypothetical IxS implementation that meets our theorem benefits from hardware improvement for all n. This is illustrated on Fig. 3 assuming a slower speedup of $2\times$, i.e., $n/2$. The OFDM's SE (Eq. 1) is also plotted across different values of n. In Section IV we study Combinadic on a practical case study to identify the inflection-point of OFDM-IM SCE on different hardwares.

IV. PRACTICAL CASE STUDY

In this section, we present a practical case study to assess the runtime IxS experiences across subblocks of different number n of subcarriers. Then, by means of our SCE metric, we identify the largest subblock size under which OFDM-IM outperforms OFDM in terms SCE (Eq. 6). As far as we know, no prior work concerns on the largest n for a real-hardware case study. This can serve as the first reference for the setup of OFDM-IM subblock size in future performance evaluation works.

We consider $N_{cp} = N/4$ and the duration of a symbol without cyclic prefix $T_N = 3.2\mu s$ in accordance with values
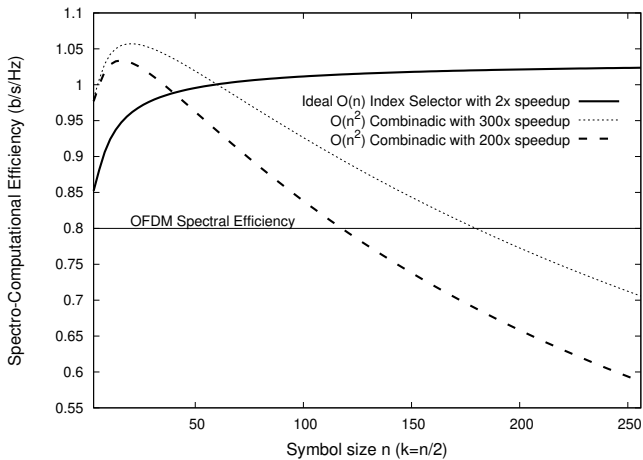
Fig. 3. Combinadic vs. ideal IxS (Theorem 1): Impact on asymptotic growth of OFDM-IM's Spectro-Computational Efficiency (Eq. 7) over hypothetical hardware speedups. Parameters: $N = n$ ($G = 1$), $N_{cp} = N/4$, $M = 2$, and $k = n/2$.
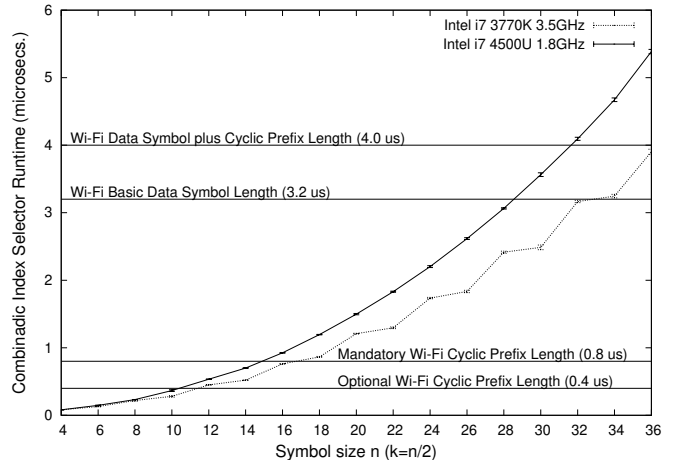

Fig. 4. Mean runtime T_α of the Combinadic algorithm [17]. Values and confidence interval length taken from Table II.

on Table 21-5 of the IEEE 802.11 standard [19]. Since an OFDM-IM transmitter requires the same computational steps of the standard OFDM, except for the IxS task, we model T_α (μs) as the mean IxS runtime. Therefore $T_\alpha = 0$ for the standard OFDM.

A. Methodology

To sample T_α, we measured the time taken by the 'Combinadic' algorithm for different values of n and $k = n/2$ (first column of Table II). We use the Stopwatch class of C# to assess the loops inside the Combinadic implementation of [17][4]. We set $k = n/2$ and assume BSPK modulation of active subcarriers (i.e., $M = 2$) to exploit the full capability of the OFDM-IM technique [3]. In each execution, we assigned our process with real-time priority and employed kernel directives to allocate one CPU core exclusively for it. We measure T_α on two distinct 64-bit CPUs, namely, Intel i7-4500U and Intel i7-3770K with clock frequencies of 1.8 GHz and 3.5 GHz, respectively. Along with core dedication, these hardware configurations have been shown to meet the real-time processing requisites of a typical OFDM Wi-Fi symbol such as FFT, equalization and interleaving [20], [21]. This way we check whether OFDM-IM outperforms OFDM under the same processing power constraints.

We generate the input for Combinadic algorithm with the Mersenne Twistter 19937 (MT) pseudo-random number generator [22]. We set up three independent instances of MT19937 with seeds 1973272912, 1822174485 and 1998078925 [23]. The three sampled T_α are averaged and forwarded to the Akaroa-2 tool for statistical treatment [24]. Akaroa-2 determines the minimum number of samples required to produce a steady-state mean estimation with a required precision. All results have a relative error below 5% and confidence interval (CI) of 95%. In all experiments the highest observed variance

was below 10^{-3}. Table II reports the mean IxS runtime (T_α) for our setups with respective OFDM-IM SCEs. For each runtime, the table also reports the half-width of the confidence interval (δ), the number x of samples needed to achieve the required precision and the number $x*$ of discarded samples before the system get at the steady-state. For the sake of space we only discuss the main values of the Table II which are T_α and SCE.

B. Discussion

Fig. 4 illustrates the mean runtime growth of Combinadic over n for two different CPUs. The values are taken from the Table II. The Fig. also contrasts the curves with lines that represent some meaningful time parameters of the IEEE 802.11 standard [19]. As one might see in the Fig. 4, the IxS runtime overhead cannot be just assumed as negligible for the overall system performance. In fact, under the typical CPU constraints of a legacy OFDM hardware, the resulting overhead of the $O(n^2)$ Combinadic algorithm is as high as some relevant time parameters of the IEEE 802.11 standard, also depicted in the figure.

Considering the mandatory timing parameters of Wi-Fi, the minimum value of n under which the IxS overhead became meaningful in our testbed was 16. In this setup, OFDM-IM maps $p_1 = \lfloor \log_2 \binom{16}{8} \rfloor = 14$ bits in all 16 indexes of the symbol. The achieved runtime for the 1.8 GHz and 3.5 GHz CPUs was about $0.92\mu s$ and $0.76\mu s$, respectively. In both cases, this is nearly the mandatory duration of cyclic prefix of Wi-Fi symbols which is $T_{cp} = 0.8\mu s$ and as high as twice the optional "short" guard interval of Wi-Fi ($T_{cp} = 0.4\mu s$) [19]. With the extra time required by the IxS algorithm, a standard OFDM Wi-Fi symbol could improve protection against inter-symbol interference [12] or compensate for the CP overhead, thus transmitting more useful bits.

In the case study $n = 32$, the IxS algorithm works to map $p_1 = \lfloor \log_2 \binom{32}{16} \rfloor = 29$ bits in 32 subblock indexes. It took about $4.09\mu s$ and $3.16\mu s$ on the 1.8 GHz and 3.5
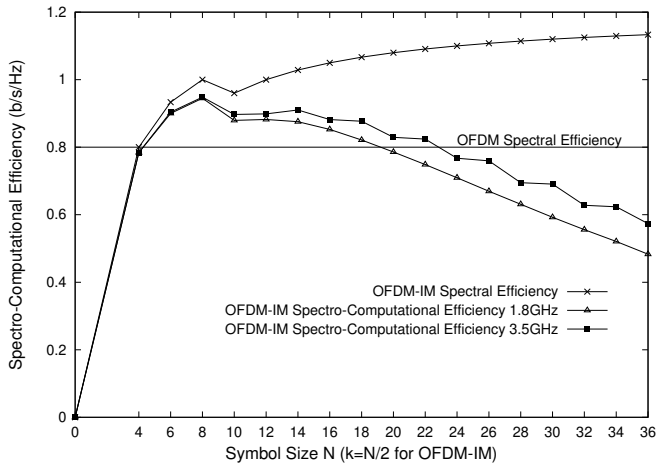
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[4]Combinadic is the method "Element" of this public implementation.

Fig. 5. Largest OFDM-IM subblock under which the computational complexity pays off for the Spectro(-computational) Efficiency. Results from column 'SCE' of Table II considering a single OFDM-IM subblock.

n	CPU (GHz)	T_α (μs)	SCE (b/s/Hz)	$\pm\delta$ (μs)	x	$x*$
4	1.8	0.813729	0.78	0.000712715	35748	5958
	3.5	0.0814348	0.78	0.0017647	56622	9437
6	1.8	0.148006	0.90	0.00194171	11166	1861
	3.5	0.129767	0.90	0.00404575	7200	1200
8	1.8	0.233303	0.94	0.00123101	10536	1756
	3.5	0.218394	0.94	0.0108699	711266	1153
10	1.8	0.366022	0.87	0.0130701	11340	945
	3.5	0.28045	0.89	0.0117626	4266	711
12	1.8	0.534652	0.88	0.00124155	4824	804
	3.5	0.450477	0.89	0.00176413	6666	1111
14	1.8	0.699524	0.87	0.00181733	6804	1134
	3.5	0.520196	0.91	0.00495158	6732	1122
16	1.8	0.92518	0.85	0.0022574	3660	610
	3.5	0.761947	0.88	0.00389986	4560	760
18	1.8	1.19477	0.82	0.00397684	3420	570
	3.5	0.864756	0.87	0.00638502	5046	841
20	1.8	1.49835	0.78	0.0094431	3276	546
	3.5	1.20886	0.82	0.00520782	3360	560
22	1.8	1.83004	0.74	0.00795553	3168	528
	3.5	1.29598	0.82	0.0123148	3078	513
24	1.8	2.20176	0.70	0.013938	3000	500
	3.5	1.73528	0.76	0.0103885	2874	479
26	1.8	2.61688	0.66	0.0134758	2730	455
	3.5	1.83402	0.75	0.0134758	3330	555
28	1.8	3.06401	0.63	0.0100115	2454	409
	3.5	2.41476	0.69	0.0164394	2634	439
30	1.8	3.56357	0.59	0.0273316	2556	426
	3.5	2.48639	0.69	0.0344689	2742	457
32	1.8	4.09465	0.55	0.0226034	2310	385
	3.5	3.16605	0.62	0.0159692	2400	400
34	1.8	4.67299	0.52	0.026905	2244	374
	3.5	3.2459	0.62	0.0261212	2256	376
36	1.8	5.38654	0.48	0.0324317	2202	367
	3.5	3.90204	0.57	0.0384168	3594	599

GHz CPUs, respectively. These resulting overheads are specially meaningful because they are very close to the duration of an entire OFDM Wi-Fi symbol with and without cyclic prefix. These values are $4.0\mu s$ and $3.2\mu s$, respectively [19]. Fig. 5 compares OFDM-IM and OFDM in terms of spectro-computational efficiency (SCE) over increasing n with values taken from Table II. The Fig. illustrates the inflection point of the OFDM-IM trade-off. In other words, we are interested in the largest subblock size under which the IxS computation overhead pays off for the overall spectral efficiency. Since both PHY layer designs have the same computational resources (unless for the IxS task of OFDM-IM), we set the extra runtime overhead of OFDM with zero ($T_\alpha = 0$). Therefore the OFDM'S SCE corresponds to the classical SE formula (Eq. 2). The IxS runtime overhead (T_α) are on Table II.

From Fig. 5, one can see that *an IxS implementation that fails our Theorem 1* represents a non-negligible overhead for the overall OFDM-IM's spectro(-computational) efficiency. In particular, assuming $k = n/2$, the Combinadic algorithm has complexity $O(n^2)$ which causes the spectro-computational efficiency of OFDM-IM to tend to zero over n. As a side note, this complexity is even higher than the iFFT's, which is widely-known to present $O(n \log_2 n)$ asymptotic cost. Thus, for sufficiently large n, IxS becomes a bottleneck along the pipeline process of the OFDM-IM baseband. Also, a hardware improvement that does not scale on n (from 1.8 GHz to 3.5 GHz in the experiment) has negligible impact to mitigate this problem.

Fig. 5 also shows that the classic SE formula is not fair nor accurate. SE predicts that OFDM-IM SE grows arbitrarily over n because it fails to incorporate the IxS's runtime overhead. When this overhead is accounted, the largest n under which OFDM-IM spectro(-computational) efficiency outperforms OFDM is 22. Above this value the smaller runtime overhead of IxS is about $1.7\mu s$ (for $n = 24$ in Table II), almost half symbol duration. Thus, the overhead does not payoff for

the transmission of $\lfloor 12 + \log_2 \binom{24}{12} \rfloor$ bits. For our tests, the best balance between bit gain and computational complexity was achieved between $\binom{6}{3}$ and $\binom{14}{7}$. For larger n, the $O(n^2)$ computational complexity of IxS neutralizes the bit gain such that OFDM-IM'S SCE decays, as one can see in Figure 5.

V. LESSONS, RECOMMENDATIONS AND FUTURE DIRECTIONS

In this work we presented theoretical and practical studies about the impact of the index selector task on the OFDM-IM spectral efficiency. To support our studies, we proposed the Spectro-Computational Efficiency (SCE) metric. SCE enabled us to parameterize the extra computational resources consumed by OFDM-IM with respect to OFDM. For the practical case study, we considered a classic implementation of IxS called Combinadic algorithm [17]. Then we parameterized the mean runtime of Combinadic under typical CPU processing con-

straints of OFDM. For the theoretical study, we characterized the computational complexity an arbitrary IxS implementation shall satisfy to ensure its asymptotic runtime overhead does not neutralize the bit gain of index modulation.

We conclude that the IxS asymptotic complexity cannot be neglected unless it meets the bounds we reported (or the hardware processing capabilities scale over n and k). In particular, we showed that the IxS computational complexity shall be bounded by $O(n)$ if $k = n/2$. Otherwise, if the IxS overhead is $\omega(n)$, the OFDM-IM SCE tends to zero over n. This is the case of the Combinadic algorithm [17] (employed in [2]–[7], [13], [14]) whose complexity is $O(n^2)$ if $k = n/2$. We also verify that setting k with lower values (e.g., $\sqrt{n}$) require more efficient IxS algorithms to compensate for the reduction in the bit gain. Hence, in future work we intend to investigate the design of a linear-time IxS for OFDM-IM with $k = n/2$. Beside, we also plan to study the SCE of other variants of OFDM-IM e.g., MM-OFDM-IM [8], [9].

In our practical case study, we employed SCE to find out the largest subblock size in which Combinadic leads OFDM-IM to outperform OFDM on different hardware. We identified that this happened for $n = 22$, $k = 11$ but, because of the IxS runtime overhead, the best SCE was achieved between $\binom{n}{k} = \binom{6}{3}$ and $\binom{14}{22}$. A future practical comparative case study consists of figuring out how to speed up the OFDM signal processing on a hardware with processing capabilities tailored for OFDM-IM. One can also investigate whether OFDM-IM SCE improves under sparse FFT (sFFT) algorithms. sFFT is usually employed to save time in video/image processing, where some nearly-zero coefficients can be nullified [25]. We note this matches OFDM-IM because $n - k$ coefficients are always zero. These future works exemplify how our metric can guide the design and comparative studies of novel computational-intensive PHY layer proposals.

REFERENCES

[1] P. K. Frenger and N. A. B. Svensson, "Parallel combinatory ofdm signaling," *IEEE Transactions on Communications*, vol. 47, no. 4, pp. 558–567, April 1999.

[2] E. Basar, U. Aygolu, E. PanayÄśrcÄś, and H. V. Poor, "Orthogonal frequency division multiplexing with index modulation," in *2012 IEEE Global Communications Conference (GLOBECOM)*, Dec 2012, pp. 4741–4746.

[3] R. Fan, Y. J. Yu, and Y. L. Guan, "Generalization of orthogonal frequency division multiplexing with index modulation," *IEEE Transactions on Wireless Communications*, vol. 14, no. 10, pp. 5350–5359, Oct 2015.

[4] ——, "Improved orthogonal frequency division multiplexing with generalised index modulation," *IET Communications*, vol. 10, no. 8, pp. 969–974, 2016.

[5] ——, "Orthogonal frequency division multiplexing with generalized index modulation," in *2014 IEEE Global Communications Conference*, Dec 2014, pp. 3880–3885.

[6] T. Mao, Z. Wang, Q. Wang, S. Chen, and L. Hanzo, "Dual-mode index modulation aided ofdm," *IEEE Access*, vol. 5, pp. 50–60, Feb 2017.

[7] T. Mao, Q. Wang, and Z. Wang, "Generalized dual-mode index modulation aided ofdm," *IEEE Communications Letters*, vol. 21, no. 4, pp. 761–764, April 2017.

[8] M. Wen, E. Basar, Q. Li, B. Zheng, and M. Zhang, "Multiple-mode orthogonal frequency division multiplexing with index modulation," *IEEE Transactions on Communications*, vol. 65, no. 9, pp. 3892–3906, Sept 2017.

[9] M. Wen, Q. Li, E. Basar, and W. Zhang, "Generalized multiple-mode ofdm with index modulation," *IEEE Transactions on Wireless Communications*, vol. 17, no. 10, pp. 6531–6543, Oct 2018.

[10] T. Mao, Q. Wang, Z. Wang, and S. Chen, "Novel index modulation techniques: A survey," *IEEE Communications Surveys Tutorials*, pp. 1–1, 2018.

[11] E. Basar, M. Wen, R. Mesleh, M. D. Renzo, Y. Xiao, and H. Haas, "Index modulation techniques for next-generation wireless networks," *IEEE Access*, vol. 5, pp. 16 693–16 746, 2017.

[12] J. Proakis and M. Salehi, *Digital Communications*. McGraw-Hill, 2008.

[13] E. Basar, U. Aygolu, E. PanayÄśrcÄś, and H. V. Poor, "Orthogonal frequency division multiplexing with index modulation," *IEEE Transactions on Signal Processing*, vol. 61, no. 22, pp. 5536–5549, Nov 2013.

[14] S. Gokceli, E. Basar, M. Wen, and G. K. Kurt, "Practical implementation of index modulation-based waveforms," *IEEE Access*, vol. 5, pp. 25 463–25 473, 2017.

[15] OEIS Foundation Inc., "Sequence Number A001405," *The On-Line Encyclopedia of Integer Sequences*, 2018, (entry by Charles R Greathouse IV). [Online]. Available: http://oeis.org/A001405.

[16] B. P. Buckles and M. Lybanon, "Algorithm 515: Generation of a vector from the lexicographical index [g6]," *ACM Trans. Math. Softw.*, vol. 3, no. 2, pp. 180–182, Jun. 1977.

[17] J. D. McCaffrey, "Generating the mth lexicographical element of a mathematical combination," *MSDN Library*, Jul 2004. [Online]. Available: http://msdn.microsoft.com/en-us/library/aa289166(VS.71).aspx

[18] D. E. Knuth, *The Art of Computer Programming: Combinatorial Algorithms, Part 1*, 1st ed. Addison-Wesley Professional, 2011.

[19] "IEEE standard for information technology–telecommunications and information exchange between systems local and metropolitan area networks–specific requirements - part 11: Wireless lan medium access control (mac) and physical layer (phy) specifications," *IEEE Std 802.11-2016 (Revision of IEEE Std 802.11-2012)*, pp. 1–3534, Dec 2016.

[20] K. Tan, H. Liu, J. Zhang, Y. Zhang, J. Fang, and G. M. Voelker, "Sora: High-performance software radio using general-purpose multi-core processors," *Commun. ACM*, vol. 54, no. 1, pp. 99–107, Jan. 2011.

[21] C. R. Berger, V. Arbatov, Y. Voronenko, F. Franchetti, and M. PÃijschel, "Real-time software implementation of an ieee 802.11a baseband receiver on intel multicore," in *2011 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, May 2011, pp. 1693–1696.

[22] M. Matsumoto and T. Nishimura, "Mersenne twister: A 623-dimensionally equidistributed uniform pseudo-random number generator," *ACM Trans. Model. Comput. Simul.*, vol. 8, no. 1, pp. 3–30, Jan. 1998.

[23] B. Hechenleitner and K. Entacher, "On shortcomings of the ns-2 random number generator," in *Communication Networks and Distributed Systems Modeling and Simulation (CNDS 2002*, 2002.

[24] D. McNickle, K. Pawlikowski, and G. Ewing, "AKAROA2: A controller of discrete-event simulation which exploits the distributed computing resources of networks," in *European Conference on Modelling and Simulation, ECMS 2010, Kuala Lumpur, Malaysia, June 1-4, 2010*, 2010, pp. 104–109.

[25] A. C. Gilbert, P. Indyk, M. Iwen, and L. Schmidt, "Recent developments in the sparse fourier transform: A compressed fourier transform for big data," *IEEE Signal Processing Magazine*, vol. 31, no. 5, pp. 91–100, Sept 2014.